PhD
1939
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BOSTON UNIVERSITY

GRADUATE SCHOOL

Dissertation

AN INVESTIGATION OF THE SEPARATIONS OF THE COMPONENTS OF THE H-ALPHA COMPLEX OF HYDROGEN

by

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(B.S., Boston University, 1933; A.M., Harvard University, 1936)

submitted in partial fulfilment of the

requirements for the degree of

Doctor of Philosophy

1939

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TOPICAL OUTLINE

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INTRODUCTION WITH HISTORICAL SURVEY OF THE WORK OF OTHER INVESTIGATORS

INTRODUCTION WITH HISTORICAL SURVEY OF THE WORK OF OTHER INVESTIGATORS

There are two hydrogen spectra —— the molecular and atomic. The latter consists of a number of lines that fall into separate series. Those known are the Lyman, Balmer, Paschen, Brackett and Pfund series ranging from the ultra violet to the infra red respectively.

The first four lines of the Balmer series H_α, H_β, H_γ and H_δ have been investigated the most since they lie in the region of the visible spectrum. Line $H\alpha$ lies in the red, H_β in the blue-green and H_γ and H_δ in the violet. The wavelengths of the individual lines of the series are governed by a simple numerical relation discovered first by Balmer in 1885 ; hence the name of the series. A complete treatment of this series is given by Sommerfeld.[1]

In 1887, Michelson and Morley were the first to discover that the line H_α was of a complex character. From examination of the visibility of the fringes of H_α formed by a Michelson interferometer, they concluded that H_α had a doublet structure. This doublet interval has been of the greatest concern even to the present day. The magnitude of the interval as determined by many investigators has differed widely and not until the past few years has a fair agreement been reached by workers in this field.

Bohr, in 1913, developed a theory for the hydrogen

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[1] Sommerfeld, Atombau and Spektrallinen, p. 344.

atom based on quantized circular orbits. Transitions of an electron from one of these orbits of quantum number 3 to another of quantum number 2 resulted in the emission of H_α. This simple theory, however, yielded no such structure to H_α as was found experimentally. Later with the refinements of the quantum theory and the introduction of relativistic wave mechanics a consistent picture of the complex structure of H_α was reached. The relativity correction[1] and the spinning electron correction introduced by Goudsmit and Uhlenbeck[2] result in five components for H_α.

Heisenberg and Jordan[3], Darwin[4], and Dirac[5] have derived expressions for the separation and relative intensities of these components. The relative intensities have been calculated by Sommerfeld and Unsold[6], Saha and Banerji[7], and Kupper[8].

Since Michelson and Morley first observed the H_α "doublet" in 1887 intensive research had been carried on for the determination of this "doublet" separation. An extensive summary of the results of the numerous investigators in this field from 1887 to 1922 has been given by G. M. Shrum[9].

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1 Sommerfeld and Unsöld, Zeits. f. Physik, 36, 259 (1926). 38, 237 (1926).

2 Goudsmit and Uhlenbeck, Physica, 6, 273 (1926).

3 Heisenberg and Jordan, Zeits. f. Physik, 37, 263 (1926).

4 Darwin, Proc. Roy. Soc., A 116, 227 (1927).

5 Dirac, Proc. Roy. Soc., A 117, 610 (1928).

6 Sommerfeld and Unsöld, Zeits, f. Physik, 38, 237 (1926).

7 Saha and Banerji, Zeits, f. Physik, 68, 704 (1931).

8 Kupper, Ann. d. Physik, 86, 511 (1928).

9 Shrum, G. M., Proc. Roy. Soc., A 105, 262 (1924).

Values of the "doublet" separation differed widely and no evidence was given at this time of the presence of the three minor components as predicted by theory.

In 1925 Janicki and Lau[1] obtained microphotometer curves of the structure of H_α as produced by a Lummer plate. Lack of sufficient asymmetry and too high a saddle height failed to reveal other components.

Houston[2] in 1926 used a Fabry-Perot interferometer instead of the Lummer-Gehrke plate. With the plates used the separation between the two lines of the "doublet" was usually less that one half of the distance between adjacent orders; thus measurements that were made visually with a comparator gave too high a value of separation while, with microphotometer curves, correction had to be made in the opposite direction for the effect of one component on the other. These disadvantages were eliminated with the Fabry-Perot interferometer, the dispersion could be chosen so as to make one component fall halfway between the adjacent orders of the other. Houston's results of the "doublet" interval were in partial agreement with previous investigators.

The most extensive paper up to this time was published by Kent, Taylor, and Pearson[3] in 1927. Two crossed Lummer plates and an echelon grating were used as resolving instruments.

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1 Janicki and Lau, Zeits. f. Physik, 35, 1(1925).

2 Houston, V. V., Astrophysical Journal, 64, 81 (1926).

3 Kent, Taylor and Pearson, Phys. Rev., 30, 266 (1927).

The resolving powers of the large plate and the echelon were in general about twice that of instruments used by others. The patterns obtained both by the crossed Lummer plates and the echelon were measured visually by the Gaertner comparator and mechanically by a microphotometer. The microphotometer curves of the crossed Lummer plate system revealed a third component which though unresolved was unquestionably present. The curves in general were ragged and irregular for the microphotometer slit had to be very short.

In 1934 Williams and Gibbs[1] discussed the methods employed in applying corrections to an observed "doublet" interval to find the true interval and relating the position of an intensity maximum to that of the components. The fine structure of H_α was analyzed by a Zeiss three-prism spectrograph together with a Fabry-Perot etalon. From an analysis of their curves they obtained $\Delta\nu = 0.308$ cm^{-1} as the separation of the main components. This interval between the main components is about 6% below the expected theoretical value $\Delta\nu = 0.328$ cm^{-1}. Their value, however is in agreement with the results of Houston and Hsieh[2]. The average of the intervals between the intensity peaks of the complex curve given by Kent, Taylor and Pearson from direct measurements of their microphotometer curves, before the "shrinking correction" was erroneously applied, was also found to be in close agreement with their values of this interval.

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[1] Williams and Gibbs, Phys. Rev., 45, 475 (1934).

[2] Houston and Hsieh, Phys, Rev., 45, 132A (1934).

Spedding, Shane and Grace[1] have also investigated the fine structure of H_α with an etalon interferometer together with an auxiliary 30 foot grating spectrograph.

Williams[2] in 1938 worked with both H_α and D_α. The microphotometer curves obtained from patterns produced by the same spectroscopic equipment of previous investigation showed component 3 partially resolved for D_α. The separation of component 2 and 3 as well as their intensities were thus more accurately determined. These values were then applied to the study of the H_α complex on the hypothesis that the interval between components 2 and 3 is the same for both H_α and D_α. It was found that component 3 is actually considerably removed from its theoretical position and its intensity is far greater than given theoretically. This then would account for the disagreement of the experimental "doublet" separation with the theoretical, since correction to the peak to peak interval of the H_α complex, based on the theoretical position and intensity of component 3, will differ from the correction based on its observed position and intensity.

Recently in a personal letter to Prof. Kent, Houston of the California Institute of Technology stated that one of his students had resolved the third component. This was done with an electrodeless discharge. The results of his work have not as yet been published. Without a doubt the outcome of this investigation with the isolation of the third component will yield the best test to date of our theoretical picture of the fine structure of H_α.

[1] Spedding, Shane and Grace, Phys. Rev., 38 (1935).

[2] Williams, Phys. Rev., 54, 558 (1938).

OBJECT OF THE RESEARCH

The preliminary investigation of W. H. Robinson[1], in the study of H-alpha of hydrogen with the use of a Lummer plate as an auxiliary spectrograph appeared very promising. This method of using two Lummer plates in tandem with a plane reflection grating located between the two Lummer plates to single out separate lines furnished a new means by which lines of a composite nature could be studied. The whole spectroscopic system had a greater dispersion than instruments used by other investigators and had the advantage that a long and narrow microphotometer slit could be used since the patterns produced were composed of straight lines.

Up to the time of the beginning of this paper no other investigators have completely isolated the third component of the theoretical five component structure of H-alpha. Results of Robinson's work showed the existence of the third component and definitely indicated the presence of the fourth. Recently in a letter to Professor Kent, Houston of the California Institute of Technology, stated that one of his students had resolved the third component, this being done by means of an electrodeless discharge as a source.

The aim of the present research was to extend the ideas advanced by Professor Kent and partially carried out by Robinson. It was hoped that with improved conditions and rearrangement of the spectroscopic system there could be obtained better photographic plates and thus better microphotometer

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1 Robinson, Ph.D. Thesis B.U.Grad.School,1937

curves which might reveal both the third and fourth components, especially the latter. With this in mind a third Lummer plate was bought for use in this problem.

The spectroscopic system finally decided upon was one in which the two Lummer plates were set directly in tandem with no intervening instruments. From a study of the relative positions of these plates, their constants, and the angles of incidence and emergence of the light rays it was hoped that by means of microphotometer patterns the wavelengths and intensities of the component lines of H-alpha could be obtained.

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FINE STRUCTURE OF H-ALPHA

DEVELOPMENT OF THEORY

The hydrogen atom consists of a proton and an electron. Due to the Coulombian field of their electrical charges, the nucleus with a charge + e and the electron with a charge - e attract each other.

The wave equation for a conservative Newtonian dynamical system is

$$\nabla^2 \psi + \frac{8\pi^2}{h^2}(W - V)\psi = 0 \qquad \text{------(1)}$$

W here represents the energy constant, and V the potential energy expressed as a function of the coordinates.

One condition imposed on this equation is that ψ must be everywhere continuous, single valued, and finite. In a given system there are only certain functions, called "eigen-functions", that satisfy the above condition and correspondingly, there are only certain characteristic values of W, the energy constant.

If in the Cartesian system we let x_1, y_1, z_1 be the position coordinates for the nucleus and x_2, y_2, z_2 those for the electron---and m_1 and m_2 be the masses, respectively, the above wave equation takes the form

$$\frac{1}{m_1}\left(\frac{\partial^2 \psi_T}{\partial x_1^2} + \frac{\partial^2 \psi_T}{\partial y_1^2} + \frac{\partial^2 \psi_T}{\partial z_1^2}\right) + \frac{1}{m_2}\left(\frac{\partial^2 \psi_T}{\partial x_2^2} + \frac{\partial^2 \psi_T}{\partial y_2^2} + \frac{\partial^2 \psi_T}{\partial z_2^2}\right) + \frac{8\pi^2}{h^2}(W_T - V)\psi_T = 0 \qquad \text{-------(2)}$$

V, the potential energy of the system, $= -\frac{Ze^2}{r}$ where r is the distance from the nucleus to the electron.

Thus $$V = \frac{-Ze^2}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

The subscript T (signifying total) is used with ψ and W to indicate that these quantities refer to the complete system, with six coordinates.

Equation 2 can be transformed into two separate equations, one which represents the translation motion of the atom as a whole and the other the relative motion of the two particles. This transformation may be effected by shifting to polar coordinates.

If x, y, and z are the Cartesian coordinates of the center of mass of the system, and r, θ, and ϕ the polar coordinates of the second particle relative to the first, the related equations are

$$x_1 = x - \frac{\mu}{m_1} r \sin\theta \cos\phi$$

$$x_2 = x + \frac{\mu}{m_2} r \sin\theta \cos\phi$$

$$y_1 = y - \frac{\mu}{m_1} r \sin\theta \sin\phi$$

$$y_2 = y + \frac{\mu}{m_2} r \sin\theta \sin\phi$$

$$z_1 = z - \frac{\mu}{m_1} r \cos\theta$$

$$z_2 = z + \frac{\mu}{m_2} r \cos\theta$$

Introducing these new variables in equation 2, we have :-

$$\frac{1}{m_1+m_2}\left(\frac{\partial^2\psi_T}{\partial x^2}+\frac{\partial^2\psi_T}{\partial y_2}+\frac{\partial^2\psi_T}{\partial z^2}\right)+\frac{1}{\mu}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_T}{\partial r}\right)\right.$$

$$\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_T}{\partial\phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi_T}{\partial\theta}\right)\right\}$$

$$+\frac{8\pi^2}{h^2}\left\{W_T - V(r,\theta,\phi)\right\}\psi_T = 0 \qquad \text{----------(3)}$$

If ψ_T is placed equal to the product of a function of x, y, z and a function of r, θ, ϕ ;

$$\psi_T(x, y, z, r, \theta, \phi) = F(x, y, z)\,\psi(r, \theta, \phi)$$

the equation can be separated into two parts, namely

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} + \frac{8\pi^2(m_1+m_2)}{h^2} W_{tr} F = 0 \qquad \text{----------(4)}$$

and $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial \phi^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{8\pi^2\mu}{h^2}\left\{W - V(r, \theta, \phi)\right\}\psi = 0 \qquad \text{----------(5)}$$

in which W_{tr} represents the translational energy of the system and W the residual energy.

If V is now a function of r alone ($V = \frac{-Ze^2}{r}$) and ψ is replaced by the product of a function of r alone, one of θ alone and one of ϕ alone,

$$\psi = R(r) \cdot \Theta(\theta) \cdot \Phi(\phi)$$

then equation 5 can be separated into three distinct total differential equations :

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = -\alpha = -m^2 \qquad \text{----------(6)}$$

$$\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) - \frac{m^2}{\sin^2\theta} = -\beta \qquad \text{----------(7)}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{\beta}{r^2}R + \frac{8\pi^2\mu}{h^2}\left\{W - V(r)\right\}R = 0 \qquad \text{----------(8)}$$

Equation 6 has eigenfunctions[1] only when α is equal to m^2, with m having any $\mp$ integral value. With these values of the parameter, m, equation 7 has acceptable solutions only when β equals $l(l+1)$ where l has any positive integral value equal to or greater than the absolute value of m. Finally, introducing values of β into equation 8 acceptable solutions are given only for certain values of W, namely --

$$W_n = -\frac{2\pi^2 \mu Z^2 e^4}{n^2 h^2} = -\frac{R_H hc Z^2}{n^2} \qquad \text{---------(9)}$$

in which n can assume integral values $l+1, l+2, \text{----} \infty$ and where

$$R_H = \frac{2\pi^2 \mu e^4}{h^3 c}$$

Solutions of equations 6, 7 and 8 listing the eigenfunctions corresponding to given values of n, l, and m are given in the references[1] at the bottom of the page.

The allowed quantum numbers for the independent wave functions can be written as follows ---

total quantum number $n = 1, 2, 3, \text{-----},$

azimuthal quantum number $l = 0, 1, 2, \text{-----}, n-1,$

magnetic quantum number $m = -l, -l+1, \text{-----}, -1, 0, +1,$
$\text{-----}, +l-1, +l.$

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[1] Schrodinger, Ann. d. Phys., 79, 361 (1926).
Pauling and Goudsmit, Structure of Line Spectra, p 27, sec. 6b.
Pauling and Wilson, Introduction to Quantum Mechanics, pp 117-124.

The above expression for the energy levels does not account for the line structure in the lines of the atomic spectrum. For a complete picture two correction terms must be made to the equation 9. The first to be considered is that due to the electron spin.

Consider the electron moving with a velocity (v) in the electric field (F) of the nucleus. The moving electron thus produces according to the electromagnetic theory a magnetic field (H) at the electron.

$$H = \frac{1}{c} F \times v$$

If the effective nuclear charge is Ze the field at the electron is

$$F = \frac{Ze}{r^3} \bar{r} \qquad \bar{r} = \text{radius vector}$$

The angular momentum thus is

$$m_0 \bar{r} \times v = \bar{l} \frac{h}{2\pi}$$

$$\therefore H = \bar{l} \times \frac{h}{2\pi m_0 c} \cdot \frac{Ze}{r^3}$$

The magnetic electron in a magnetic field undergoes a precession about the field direction, with an angular velocity equal to the product of the field strength (H) and the ratio of the magnetic $(2\bar{s}\frac{h}{2\pi} \cdot \frac{e}{2m_0 c})$ and the mechanical moment $(\bar{s}\frac{h}{2\pi})$ of the spinning electron

$$w_H = \frac{e^2}{m_0^2 c^2} \cdot \frac{Z}{r^3} \cdot \frac{\bar{l} h}{2\pi}$$

From the work of L. H. Thomas[1] in the relativ-

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[1] L. H. Thomas, Nature, 117, 514 (1926).

istic treatment of the motion of the spinning electron there exists another precession of the electron axis besides the above magnetic precession. This relativistic precession (ω_T) has one half the value of the magnetic, and has the opposite sign.

The actual angular velocity of the electron axis thus becomes

$$\omega = \omega_H + \omega_T = \frac{1}{2} \frac{e^2}{m_o^2 c^2} \cdot \frac{Z}{r^3} \cdot \frac{\bar{l}\, h}{2\pi}$$

The energy of such a precessional motion is equal to the product of the angular velocity and the projection of the mechanical moment ($\bar{s} \frac{h}{2\pi}$) of the precessing system on the axis of precession; i.e., on H or $\bar{l}$:

$$W_s = \frac{1}{2} \frac{e^2 h^2}{4\pi^2 m_o^2 c^2} \cdot \frac{Z}{r^3} sl \cos(\bar{s}\bar{l}) \text{ -----(10)}$$

The factor $1/r^3$ is not constant during the motion of the electron. From quantum mechanics[1]

$$\left(\overline{\frac{1}{r^3}}\right) = \frac{64\pi^6 m_o^3 e^6 Z^4}{h^6 n^3 l (l+\frac{1}{2}) (l+1)} \text{ ----------(11)}$$

Thus equation 10 can be written in the form

$$W_s = \frac{Rhc\alpha^2 Z^4}{n^3 l (l+\frac{1}{2}) (l+1)} ls \cos(\bar{l}\bar{s}) \text{ ----------(12)}$$

where $R = \frac{2\pi^2 m_o e^4}{ch^3}$

and $\alpha = \frac{2\pi e^2}{hc}$

The total angular momentum which is given by the quantum vector $\bar{j}$, is the resultant of the orbital momentum $\bar{l}$

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[1] Pauling and Goudsmit, Structure of Line Spectra, section 7, p 32.

and the spin moment $\bar{s}$ —— where the magnitudes of these vectors in terms of quantum numbers are $\sqrt{j(j+1)}$, $\sqrt{l(l+1)}$, and $\sqrt{s(s+1)}$ respectively.

The following equation follows from a discussion of the resultant quantum vector.[1]

$$sl\cos(\bar{s}\bar{l}) = \frac{j(j+1) - s(s+1) - l(l+1)}{2}$$

Since j can assume only the values $l \mp \frac{1}{2}$

$$sl\cos(\bar{l}\bar{s}) = \mp\frac{(l+\frac{1}{2}) - \frac{1}{2}}{2}$$

$$W_s = \frac{Rhc\alpha^2Z^4}{n^3 l(l+\frac{1}{2})(l+1)} \cdot \mp\frac{(l+\frac{1}{2}) - \frac{1}{2}}{2} \qquad \text{---------(13)}$$

The second correction term to be added to the energy values is that due to the consideration of the relativistic change in mass with the velocity of the electron. This correction splits up the energy levels given by n into n levels (s, p, d, f, ---etc.). The equation is

$$W_l = \frac{Rhc\alpha^2Z^4}{n^3}\left(\frac{3}{4n} - \frac{1}{l+\frac{1}{2}}\right) \qquad \text{---------(14)}$$

which was first derived by W. Pauli[2] from quantum mechanics.

Combining equations 13 and 14, we have

$$W' = W_l + W_s = \frac{Rhc\alpha^2Z^4}{n^3}\left(\frac{3}{4n} - \frac{1}{l+1}\right) \text{ for } j = l + \tfrac{1}{2}$$

$$W' = W_l + W_s = \frac{Rhc\alpha^2Z^4}{n^3}\left(\frac{3}{4n} - \frac{1}{l}\right) \text{ for } j = l - \tfrac{1}{2}$$

Therefore total energy $W = W_n + W'$ or

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[1] Pauling and Goudsmit, Structure of Line Spectra, section 14a, p 55.

[2] Pauli, W., Z. f. Phys., 36 , 336 (1926)

$$W = -\frac{RhcZ^2}{n^2} + \frac{Rhc\alpha^2 Z^4}{n^3}\left(\frac{3}{4n} - A\right)$$

where $A = \frac{1}{l+1}$ for $j = l + \frac{1}{2}$

and $A = \frac{1}{l}$ for $j = l - \frac{1}{2}$

Certain energy levels will be found to coincide;- namely, those with the same values of j.

$$R_H = \frac{2\pi^2 \mu e^4}{ch^3} = 109677.759 \text{ cm}^{-1}, \quad \alpha^2 = 5.305 \times 10^{-5}$$

$$W = -\frac{R_H hcZ^2}{n^2} + \frac{Rhc\alpha^2 Z^4}{n^3} = \frac{109677.759 \times 6.547 \times 10^{-27} \times 2.99796 \times 10^{10}}{n^2}$$

$$+ \frac{109737.42 \times 6.547 \times 10^{-27} \times 2.99796 \times 10^{10} \times 5.305 \times 10^{-5}}{n^3}$$

The equation for the energy levels is thus

$$W = -\frac{2.1527 \times 10^{-11}}{n^2} + \frac{1.1426 \times 10^{-15}}{n^3}\left(\frac{3}{4n} - A\right)$$

Values of A are given above.

The conditions imposed on the three quantum numbers n, l, and j are

$n = 1, 2, 3, \text{--------} \infty$

$l = 0, 1, 2, \text{--------} (n-1)$

$j = l \mp \frac{1}{2}$

The main energy levels n = 3 and n = 2 for Hα are thus split up into 3l and 2l levels respectively. These in turn into 2j levels. It will be found that the energy levels for the same values of j coincide. In the figure 1 those which coincide are slightly separated for clearness. In all we have but five different energy levels. The large letters (S, P, D, F, ---) correspond to values of l = 0, 1, 2, ---etc.

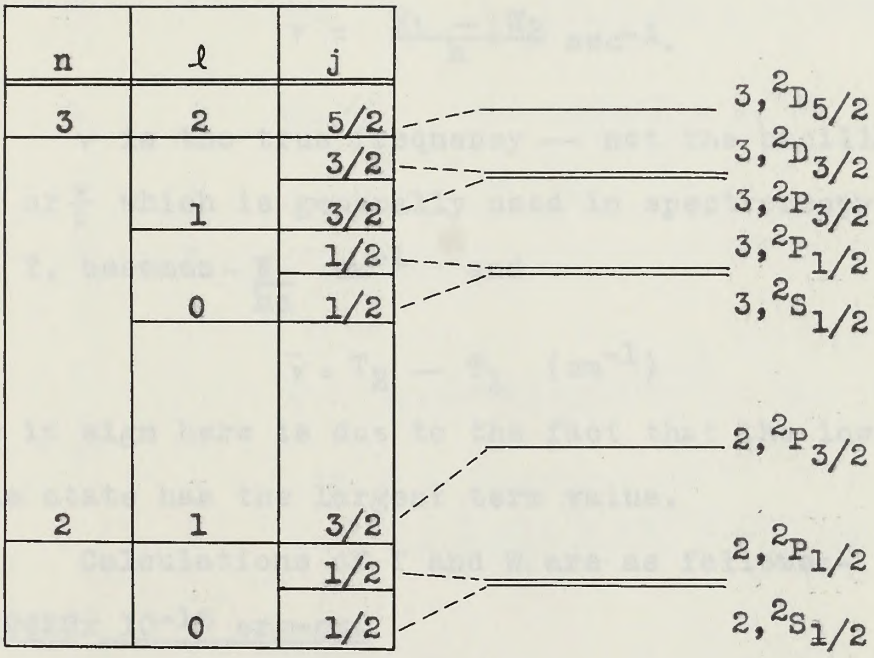

Figure I

ENERGY LEVELS

The frequency of a line emited by radiation attending a transition from one energy state W_1 to a lower state W2 is given by

$$\nu = \frac{W_1 - W_2}{h} \text{ sec}^{-1}.$$

ν is the true frequency — not the oscillation frequency $\tilde{\nu}$ or $\frac{\nu}{c}$ which is generally used in spectroscopy. The term value, T, becomes $-\frac{W}{hc}$ cm^{-1} and

$$\bar{\nu} = T_2 - T_1 \quad (\text{cm}^{-1})$$

The change in sign here is due to the fact that the lowest and most stable state has the largest term value.

Calculations of T and W are as follows:-

$hc = 1.9628 \times 10^{-16}$ erg-cm.

For $2,^2S_{1/2}$ $\left|\begin{array}{l} n = 2 \\ \ell = 0 \\ j = \frac{1}{2} \end{array}\right.$

$$W = -\frac{2.1527 \times 10^{-11}}{4} + 1.1426 \times 10^{-15}\left(\frac{3}{8} - 1\right)$$

$$W = -5.382 \times 10^{-12} - 0.8927 \times 10^{-16}$$

$$T = -\frac{M - 0.8927 \times 10^{-16}}{hc} = A + .45481 \text{ cm}^{-1}$$

also $2,^2P_{1/2}$ $\left|\begin{array}{l} n = 2 \\ \ell = 1 \\ j = \frac{1}{2} \end{array}\right.$

where $M = -5.382 \times 10^{-12}$

$$A = -\frac{M}{hc}$$

$2,^2P_{3/2}$ $\left|\begin{array}{l} n = 2 \\ \ell = 1 \\ j = \frac{3}{2} \end{array}\right.$

$$W = M + \frac{1.1426 \times 10^{-15}}{8}\left(\frac{3}{8} - \frac{1}{2}\right)$$

$$W = M - 0.1785 \times 10^{-16}$$

$$T = -\frac{M - .1785 \times 10^{-16}}{hc} = A + .09094 \text{ cm}^{-1}$$

$3,{}^2S_{1/2}$ | $n = 3$, $\ell = 0$, $j = \frac{1}{2}$

$$W = -\frac{2.1527 \times 10^{-11}}{9} + \frac{1.1426 \times 10^{-15}}{27}\left(\frac{1}{4} - 1\right)$$

$$= -2.3919 \times 10^{-12} - 3.1739 \times 10^{-17}$$

$$T = -\frac{G - 3.1739 \times 10^{-17}}{hc} = B + .16170 \text{ cm}^{-1}$$

where $G = -2.3919 \times 10^{-12}$

$$B = -\frac{G}{hc}$$

also

$3,{}^2P_{1/2}$ | $n = 3$, $\ell = 1$, $j = \frac{1}{2}$

$3,{}^2D_{3/2}$ | $n = 3$, $\ell = 1$, $j = \frac{3}{2}$

$$W = G + \frac{1.1426 \times 10^{-15}}{27}\left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= G - 1.0580 \times 10^{-17}$$

$$T = -\frac{G - 1.058 \times 10^{17}}{hc} = B + .053902 \text{ cm}^{-1}$$

also

$3,{}^2P_{3/2}$ | $n = 3$, $\ell = 2$, $j = \frac{3}{2}$

$3,{}^2D_{5/2}$ | $n = 3$, $\ell = 2$, $j = \frac{5}{2}$

$$W = G + \frac{1.1426 \times 10^{-15}}{27}\left(\frac{1}{4} - \frac{1}{3}\right)$$

$$= G - .35265 \times 10^{-17}$$

$$T = -\frac{G - .35265 \times 10^{-17}}{hc} = B + .017967 \text{ cm}^{-1}$$

These values of T are listed in table I, page 23.

The selection rule governing the possible transitions of an electron from energy state to another is that j can change only by 1 or 0, and any violation of this rule is attributed to the presence of external electric or magnetic fields[1]. The total quantum number n can change by any integral value.

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[1] White, Introduction to Atomic Spectra, pp149-170; 401-417.

The possible transitions arising from the selection rules are represented schematically in figure 2, page 24. It is to be remembered that energy levels ($3,^2D_{3/2}$ and $3,^2P_{3/2}$), ($3,^2P_{1/2}$ and $3,^2S_{1/2}$) and ($2,^2P_{1/2}$ and $2,^2S_{1/2}$) coincide. Thus five lines are given, representing the fine structure of H_α.

Figure 2, page 24 also gives the separation between the lines and the relative intensities. The components are numbered in order of their descending intensity. The calculated frequencies are listed in table 1, page 23. In this table are also listed the relative intensities[1] of the permitted transitions as predicted by wave mechanics.

[1] Sommerfeld and Unsold, Zeits. f. Phys., 36, 259 (1926).
Schrodinger, E., Ann. d. Phys., 80, 437 (1926). 38, 237 (1926).
Ruark, A. E., and Urey, H. C., Atoms, Molecules and Quanta.

Line	Transitions	Freq. Differences cm^{-1}	ν cm^{-1}
5	$3,^2S_{1/2}-2,^2P_{3/2}$	A + .09094 - B - .16170	(C - 1)+.92924
4	$3,^2D_{3/2}-2,^2P_{3/2}$	A + .09094 - B - .05390	1.03704
1	$3,^2D_{5/2}-2,^2P_{3/2}$	A + .09094 - B - .01797	1.07297
3	$3,^2P_{1/2}-2,^2S_{1/2}$	A + .45481 - B - .16170	1.29311
	$3,^2S_{1/2}-2,^2P_{1/2}$	" "	"
2	$3,^2D_{3/2}-2,^2P_{1/2}$	A + .45481 - B - .05390	1.40091
	$3,^2P_{3/2}-2,^2S_{1/2}$	" "	"

Line	Intensity	Freq. cm^{-1}	$\Delta\nu\ 10^{-3} cm^{-1}$
5	2/3	.9292	108
4	256/75	1.0370	36
1	768/25	1.0730	220
3	35/9	1.2931	108
2	1088/45	1.4009	

TABLE I

Line	Transitions	Freq. Difference cm^{-1}	ν cm^{-1}
5	$^3P_1/^2P_3 - ^2P_3/^2S_1$	A + .09094 - B - .16170	(C - 1)+.92924
4	$^3P_2/^2P_3 - ^2P_3/^2S_1$	A + .09094 - B - .05390	1.03704
3	$^3P_1/^2P_3 - ^2P_3/^2S_1$	A + .09094 - B - .01797	1.07267
2	$^3P_1/^2P_1 - ^2P_3/^2S_1$	A + .45461 - B - .16170	1.29311
"	$^3P_0/^2P_1 - ^2P_3/^2S_1$	" "	"
1	$^3P_2/^2P_1 - ^2P_3/^2S_1$	A + .45461 - B - .05390	1.49071
"	$^3P_1/^2P_1 - ^2P_3/^2S_1$	" "	"

Line	Intensity	Freq. cm^{-1}	$\Delta\nu$ $10^{-3}cm^{-1}$
5	2/3	.0292	
			103
4	256/75	1.0370	
			36
1	768/25	1.0730	
			220
3	384/9	1.2931	
			103
2	1088/45	1.4009	

TABLE I

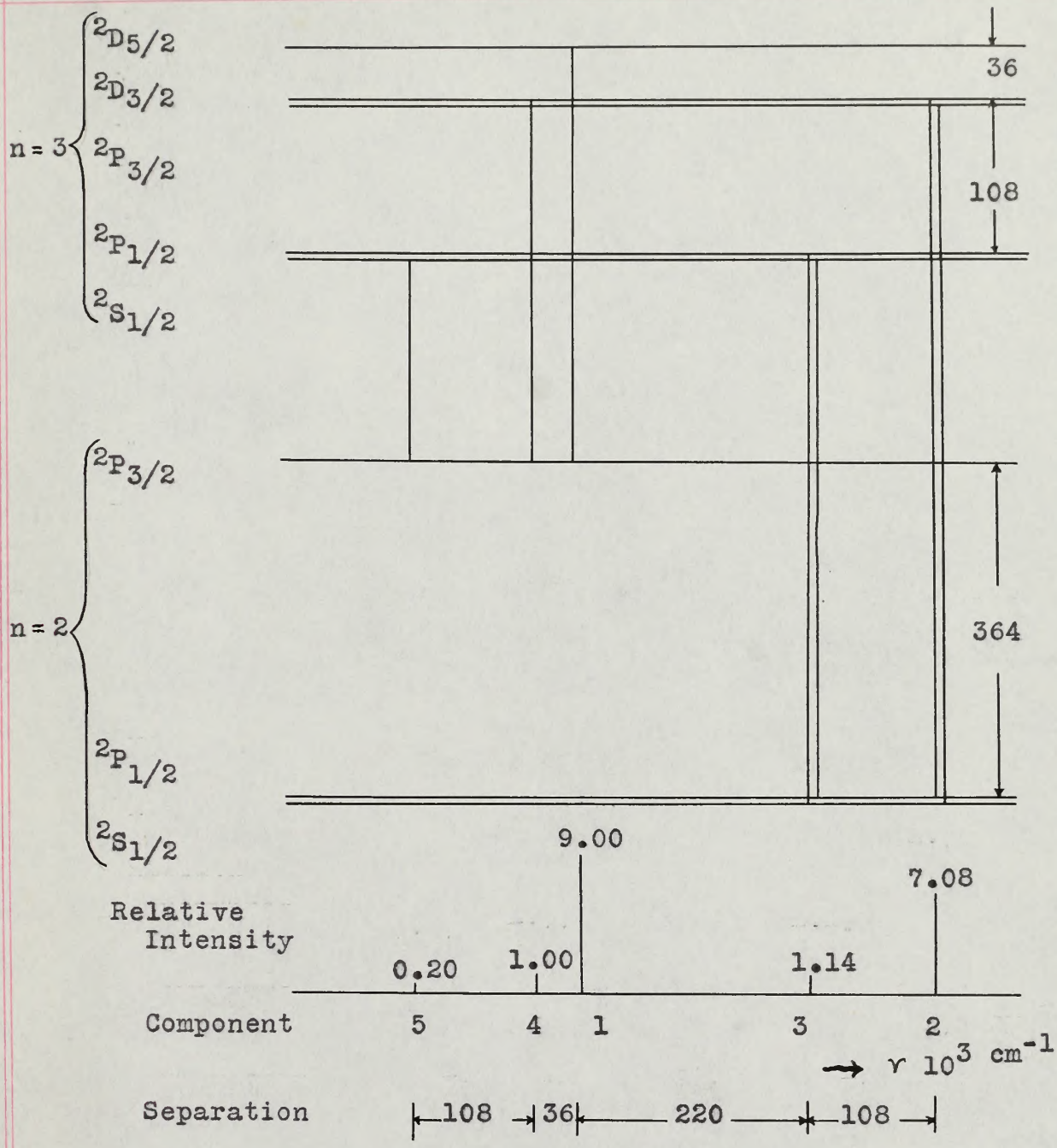
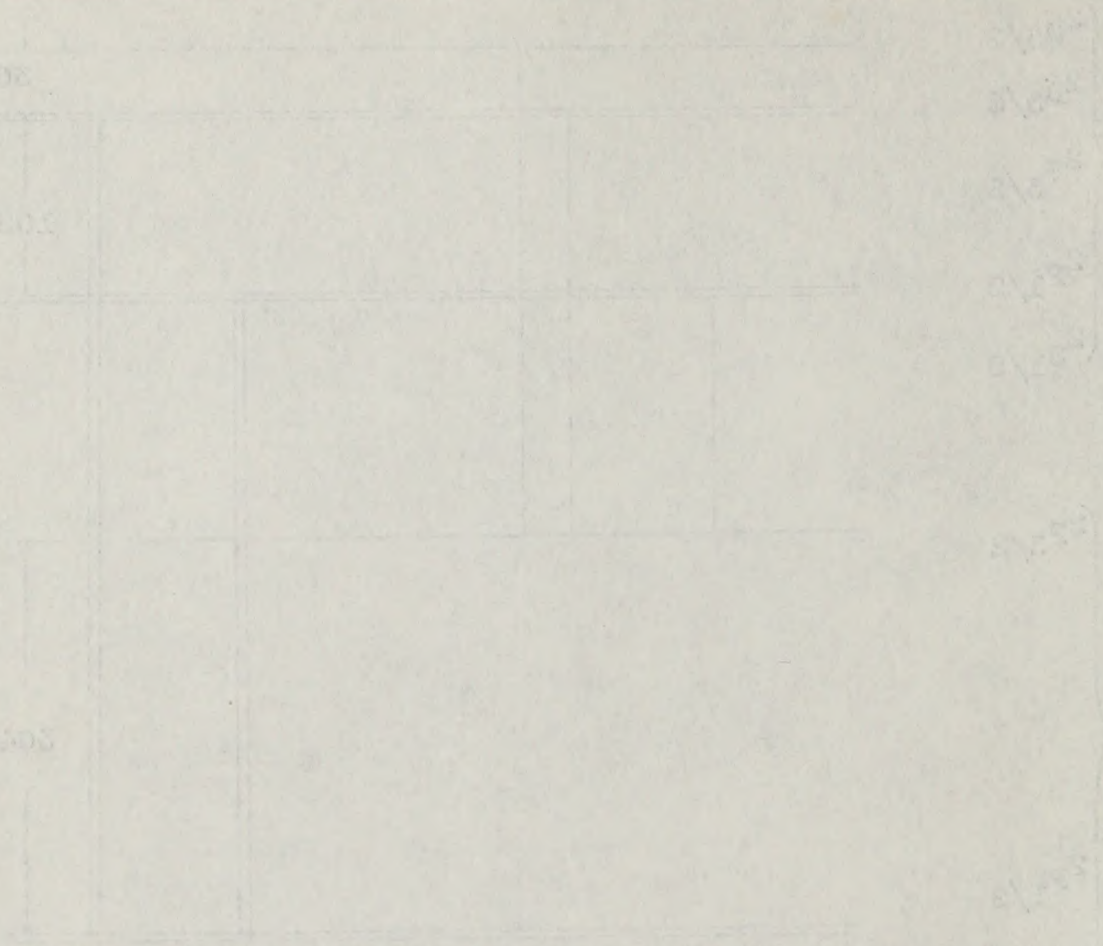


Figure 2

THEORETICAL FINE STRUCTURE OF H-ALPHA



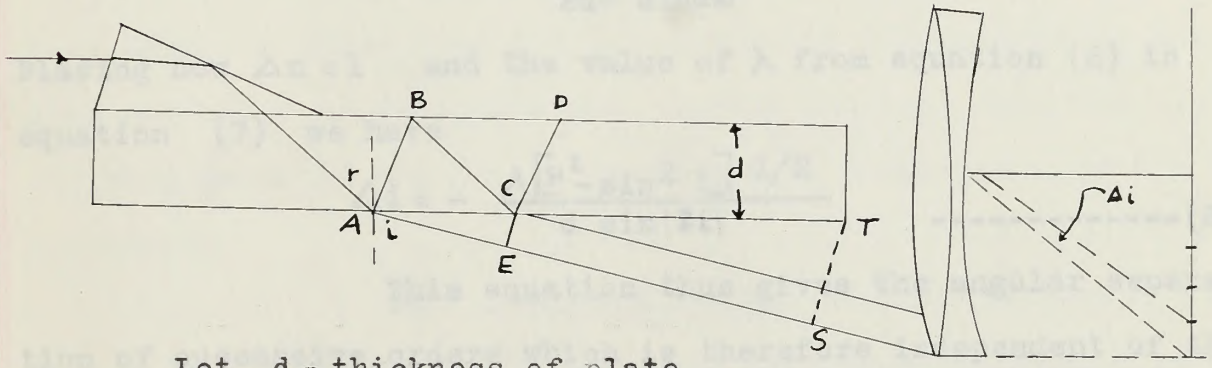
THEORY OF LUMMER PLATES

SEPARATION BETWEEN ORDERS

Consider the following diagram in which a ray of light of wavelength λ passes through a Lummer plate.

FORMATION OF FRINGES

FIG. 3



Let d = thickness of plate

μ = refractive index for wavelength λ

s = path difference $= n\lambda$

n = Order number for wavelength λ

i = angle between normal to the plate and the emergent ray

r = angle between normal to the plate and the ray within the plate

The condition for the reinforcement of the rays is

$$S = n\lambda = \mu (ABC) - AE$$

or $$n\lambda = 2d\mu \cos r \qquad \text{----------(1)}$$

From trigonometry $\cos r = \sqrt{\mu^2 - \sin^2 i}/\mu$ since $\sin i = \mu \sin r$

thus $$n\lambda = 2d\sqrt{\mu^2 - \sin^2 i} \qquad \text{----------(2)}$$

Squaring both sides we have

$$n^2\lambda^2 = 4d^2\mu^2 - 4d^2\sin^2 i \qquad \text{----------(3)}$$

Differentiating this equation with i and n as variables we have

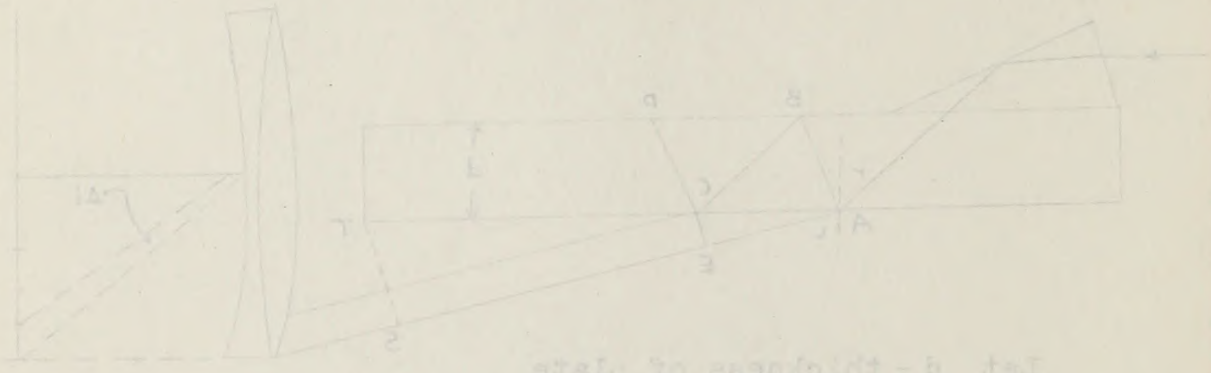
$$2n\lambda^2 \Delta n = -8d^2 \sin i \cos i \, \Delta i \qquad \text{----(4)}$$

SEPARATION BETWEEN ORDERS

Consider the following diagram in which a ray of light of wavelength λ passes through a lummer plate.

RETARDATION OF RAYS

FIG. 3



Let d = thickness of plate

μ = refractive index for wavelength λ

δ = path difference = $m\lambda$

n = Order number for wavelength λ

i = angle between normal to the plate and the emergent ray

γ = angle between normal to the plate and the ray within the plate

The condition for the reinforcement of the rays is

$$\delta = m\lambda = \mu (ABC) - AE$$

or $$m\lambda = 2d\mu\cos\gamma \quad \text{----------(1)}$$

From trigonometry $\cos\gamma = \sqrt{\mu^2 - \sin^2 i}$ since $\sin i = \mu \sin\gamma$

thus $$m\lambda = 2d\sqrt{\mu^2 - \sin^2 i} \quad \text{----------(2)}$$

Squaring both sides we have

$$m^2\lambda^2 = 4d^2\mu^2 - 4d^2\sin^2 i \quad \text{----------(3)}$$

Differentiating this equation with i and m as variables we have

$$2m\lambda^2 \Delta m = -8d^2 \sin i \cos i \Delta i \quad \text{----(4)}$$

From trigonometry $\frac{\sin 2i}{2} = \sin i \cos i$ ----------(5)

Substituting equation (5) in (4) gives

$$n\lambda^2 \Delta n = -2d^2 \cdot \sin 2i \, \Delta i \qquad \text{----------(6)}$$

and solving for Δi

$$\Delta i = - \frac{n\lambda^2}{2d^2 \sin(2i)} \cdot \Delta n \qquad \text{----------(7)}$$

Placing now $\Delta n = 1$ and the value of λ from equation (2) in equation (7) we have

$$\Delta i = - \frac{\lambda[\mu^2 - \sin^2 i]^{1/2}}{d \sin(2i)} \qquad \text{----------(8)}$$

This equation thus gives the angular separation of successive orders which is therefore independent of the length of the plate, directly proportional to the wavelength and inversely proportional to the thickness of the plate.

DISPERSION

If the equation (3) above is differentiated with respect to λ considering μ and i as variables, we have

$$n^2 \lambda = 4d^2 \mu \frac{\partial \mu}{\partial \lambda} - 2d^2 \sin 2i \frac{\partial i}{\partial \lambda} \qquad \text{----------(9)}$$

Solving this equation for $\frac{\partial i}{\partial \lambda}$ results:

$$\frac{\partial i}{\partial \lambda} = \frac{4d^2 \mu \frac{\partial \mu}{\partial \lambda} - n^2 \lambda}{2d^2 \sin(2i)} \qquad \text{----------(10)}$$

Substituting the value of $n^2\lambda$ from equation (3) in (10)

$$\frac{\partial i}{\partial \lambda} = \frac{4d^2\mu\frac{\partial \mu}{\partial \lambda} - \frac{4d^2\mu^2 - 4d^2\sin^2 i}{\lambda}}{2d^2 \sin 2i}$$

$$\frac{\partial i}{\partial \lambda} = \frac{2\lambda\mu\frac{\partial \mu}{\partial \lambda} - 2(\mu^2 - \sin^2 i)}{\lambda \sin 2i} \qquad \text{----------(11)}$$

This equation shows that the dispersion is independent of the size and shape of the plate and depends entirely upon the optical properties of the glass, the angle of emergence, and the wavelength.

RESOLVING POWER

Setting Δi of equation (10) equal to the Δi of equation (8), the resulting value of $\Delta\lambda$ is then the range of the plate without overlap, that is to say $\Delta\lambda$ is the difference of wavelength which a line in any order must have so that it coincides with the image in the next order. We have thus

$$\Delta\lambda = \frac{n\lambda^2}{n^2\lambda - 4d^2\mu\frac{\partial \mu}{\partial \lambda}} \qquad \text{----------(12)}$$

Consider Fig. 3. The width of the beam or wave front ST entering the telescope is AT cos $i = l \cos i$, l here is the length of the plate. Now the angle ∂i between the central maximum and the first minimum of a beam of this width is

$$\partial i = \frac{\lambda}{l \cos i} \qquad \text{----------(13)}$$

Rearranging equation (11) we have

$$di = -\frac{1}{\lambda \sin 1 \cos i}\left\{\mu^2 - \sin^2 i - \lambda\mu\frac{d\mu}{d\lambda}\right\}d\lambda \text{----------(14)}$$

If we make now $di = \frac{\lambda}{1 \cos i}$, the corresponding $d\lambda$ will be then the smallest wavelength change resolvable.

Solving for $\frac{\lambda}{d\lambda}$ the resolving power, we get

$$\frac{\lambda}{d\lambda} = \lambda \frac{1}{\sin i}\left\{\mu^2 - \sin^2 i - \lambda\mu\frac{d\mu}{d\lambda}\right\} \text{----------(15)}$$

This value of the resolving power, however, is only strictly correct at grazing emergence since $di = \frac{\lambda}{1 \cos i}$ only when the amplitude is uniform over the wavefront ST. To obtain a close approximation to the limit of resolving power we write sin i = 1, whence:-

$\frac{\lambda}{d\lambda} \simeq \frac{\ell}{\lambda}(\mu^2 - 1)$ Roughly then the resolving power lies between 1.4 times and twice the metrical length of the plate measured in wavelengths. The dispersion factor term which involves $\frac{d\mu}{d\lambda}$ only makes a correction of the order of 5 to 10 per cent.

TABLE II

RESOLVING POWERS OF THE LUMMER PLATES

Plate	Resolving Power $\lambda/d\lambda$
Smallest	254,000
Medium	492,800
Largest	702,000

THEORY OF TWO LUMMER PLATES IN TANDEM

In figure 4 a ray of light of wavelength λ upon entering the Lummer plate through the prism will undergo a series of internal reflections within the plate at points A, B, C, D etc. At these points a portion of the light will be refracted giving rise to two groups of parallel beams 1, 2, ----n, one on each side of the Lummer plate. These successive emergent parallel beams can be brought to a focus by an objective and if in phase will reinforce to produce bright fringes.

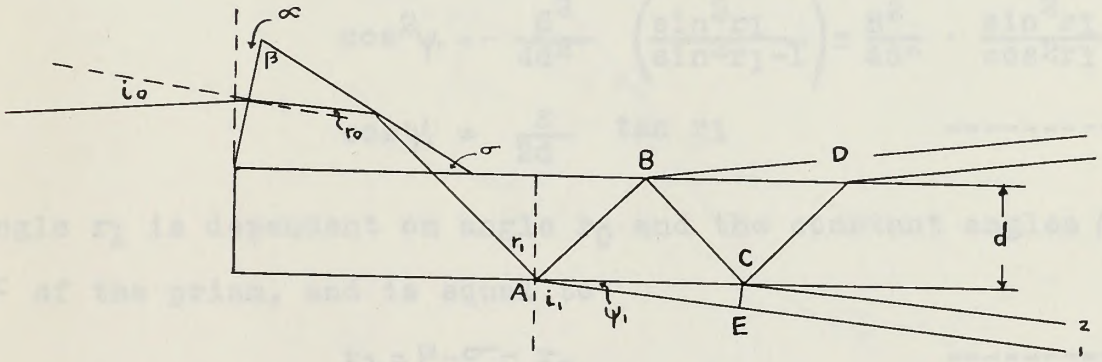


Figure 4

RAY OF LIGHT THROUGH FIRST LUMMER PLATE

Let d = thickness of plate

μ = refractive index for wavelength λ

s = path difference= $n\lambda$

n = order number for wavelength λ

The condition for reinforcement of the beams is

$$S = n\lambda = \mu\,(ABC) - AE \qquad \text{-------(1)}$$

THEORY OF TWO LUMMER PLATES IN TANDEM

In figure 4 a ray of light of wavelength λ upon entering the Lummer plate through the prism will undergo a series of internal reflections within the plate at points A, B, C, D etc. At these points a portion of the light will be refracted giving rise to two groups of parallel beams 1, 2, ----n, one on each side of the Lummer plate. These successive emergent parallel beams can be brought to a focus by an objective and if in phase will reinforce to produce bright fringes.

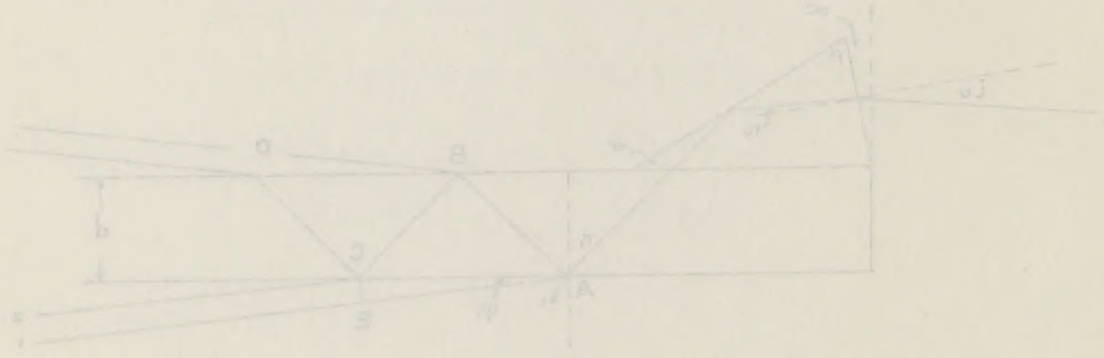


Figure 4

RAY OF LIGHT THROUGH TWO LUMMER PLATE

Let d = thickness of plate

μ = refractive index for wavelength λ

a = path difference = $n\lambda$

n = order number for wavelength λ

The condition for reinforcement of the beams is

$$a = n\lambda = \mu\,(ABC) - AE \qquad \text{--------(1)}$$

or $S = 2d\mu \cos r_1$

From trigonometry $\cos r_1 = \sqrt{\mu^2 - \sin^2 i_1}\,/\mu$ since $\sin i_1 = \mu \sin r_1$

thus $S = 2d\sqrt{\mu^2 - \sin^2 i_1}$

Let ψ_1 be the angle of emergence of the parallel beams, then

$$S = 2d\sqrt{\mu^2 - \cos^2 \psi_1}$$

Squaring both sides and solving for $\cos \psi_1$, we have

$$\cos \psi_1 = \sqrt{\mu^2 - S^2/4d^2}$$

Now $$\mu^2 = \frac{\sin^2 i_1}{\sin^2 r_1} = \frac{\cos^2 \psi_1}{\sin^2 r_1}$$

hence $$\cos^2 \psi_1 = -\frac{S^2}{4d^2} + \frac{\cos^2 \psi_1}{\sin^2 r_1}$$

$$\cos^2 \psi_1 = -\frac{S^2}{4d^2}\left(\frac{\sin^2 r_1}{\sin^2 r_1 - 1}\right) = \frac{S^2}{4d^2} \cdot \frac{\sin^2 r_1}{\cos^2 r_1}$$

$$\cos \psi_1 = \frac{S}{2d} \tan r_1 \qquad \text{----------(2)}$$

Angle r_1 is dependent on angle r_0 and the constant angles β and σ of the prism, and is equal to

$$r_1 = \beta - \sigma - r_0 \qquad \text{----------(3)}$$

thus $$\cos \psi_1 = \frac{S}{2d} \tan (\beta - \sigma - r_0) \qquad \text{----------(4)}$$

In figure 5 let

θ = the angle between the Lummer plates

i_2 = angle of incidence of the parallel rays upon the prism of the second plate

ψ_3 = angle of emergence of rays from plate

d' = thickness of plate

μ' = index of refraction of glass for wavelength λ.

The angle of incidence i_2 is dependent on ψ_1, θ and α'. From the figure below

$$i_2 = \theta + \alpha' - \psi_1 \qquad \text{and} \qquad \psi_1 = \theta + \alpha' - i_2 \qquad \text{------(5)}$$

Sustituting this value of ψ_1 in equation 4, we have

$$\cos(\theta + \alpha' - i_2) = \frac{S}{2d} \operatorname{Tan}(\beta\sigma - r_o) \qquad \text{------(6)}$$

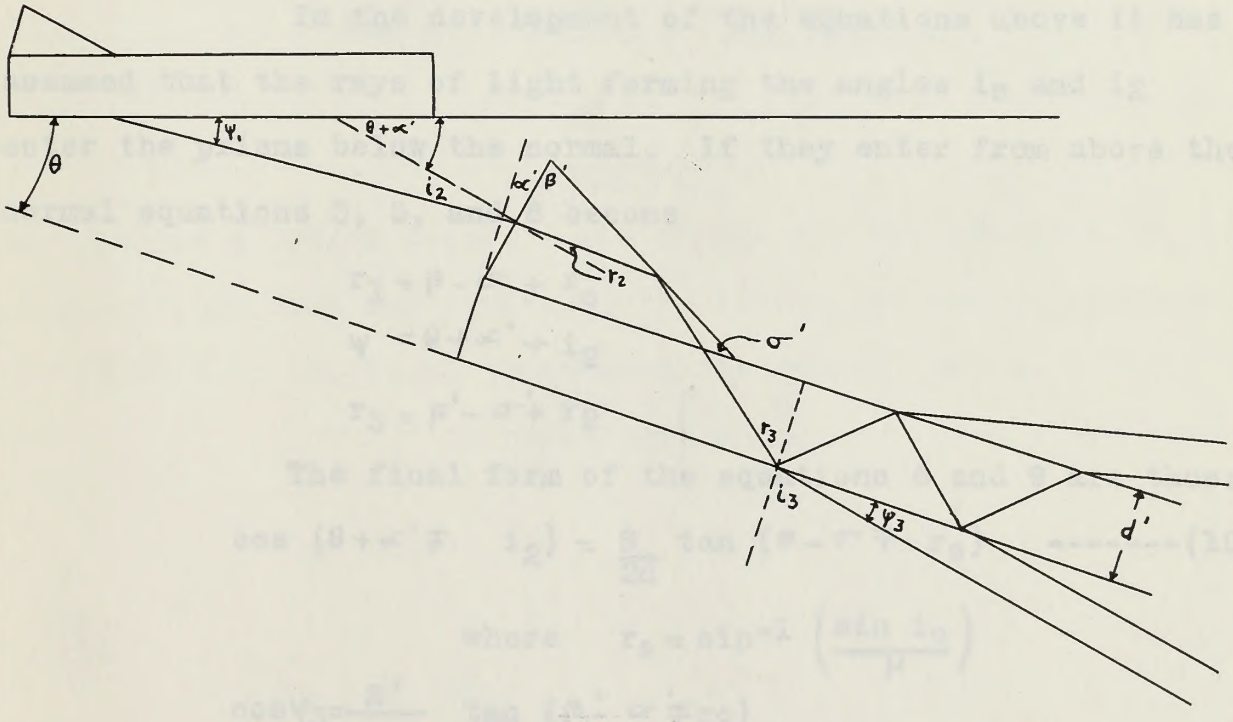


Figure 5

RAY OF LIGHT THROUGH SECOND LUMMER PLATE

A treatment of the second plate as the first leads to equations similar to 2, 3 and 4 namely

$$\cos\psi_3 = \frac{S'}{2d'} \tan r_3 \qquad \text{----------(7)}$$

$$r_3 = \beta' - \sigma' - r_2 \qquad \text{----------(8)}$$

$$\cos\psi_3 = \frac{S'}{2d'} \tan(\beta' - \sigma' - r_2) \qquad \text{----------(9)}$$

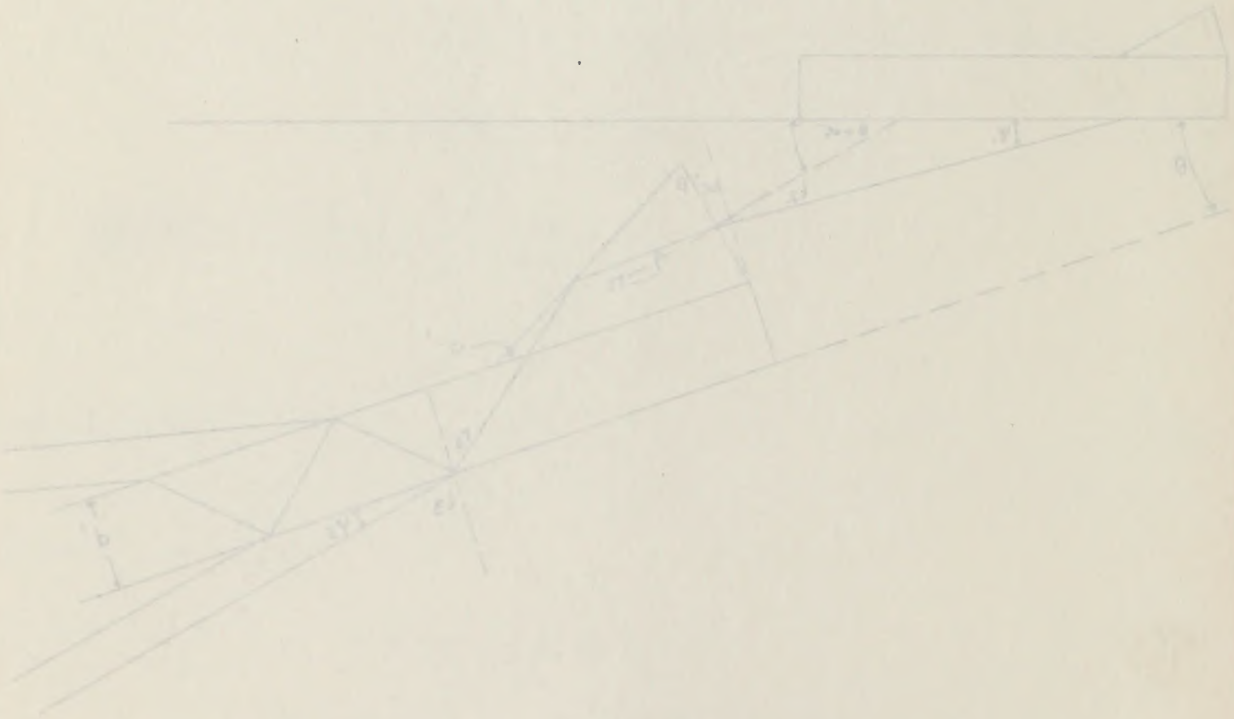


Figure 5

Given now the angle of incidence i_o, the angle of refraction r_o can be found from the relation $\sin r_o = (\mu - 1)\sin i_o$. Equation 6 will then lead to values of i_2, angles of incidence on the second plate, for different values of the order number n. From these values of i_2, the angles of r_2 can be found from the relation $\sin r_2 = (\mu' - 1)\sin i_2$ and finally values of ψ_3 are calculated by equation 9 for the different values of n'.

In the development of the equations above it has been assumed that the rays of light forming the angles i_o and i_2 enter the prisms below the normal. If they enter from above the normal equations 3, 5, and 8 become

$$r_1 = \beta - \sigma + r_o$$
$$\psi = \theta + \alpha' + i_2$$
$$r_3 = \beta' - \sigma' + r_2$$

The final form of the equations 6 and 9 are thus:-

$$\cos(\theta + \alpha' \mp i_2) = \frac{S}{2d} \tan(\beta - \sigma \mp r_o) \quad \text{-------(10)}$$

where $r_o = \sin^{-1}\left(\frac{\sin i_o}{\mu}\right)$

$$\cos\psi_3 = \frac{S'}{2d'} \tan(\beta' - \sigma' \mp r_2)$$

where $r_2 = \sin^{-1}\left(\frac{\sin i_2}{\mu'}\right)$ -------(11)

Minus signs are to taken when the rays of light enter the prisms from below the normal and plus signs when the rays enter from above.

If the successive emergent beams for a given value

of Ψ_3, which are parallel, are brought to a common focus by an objective at a point P on the focal plane they will reinforce

POSITION OF FRINGES

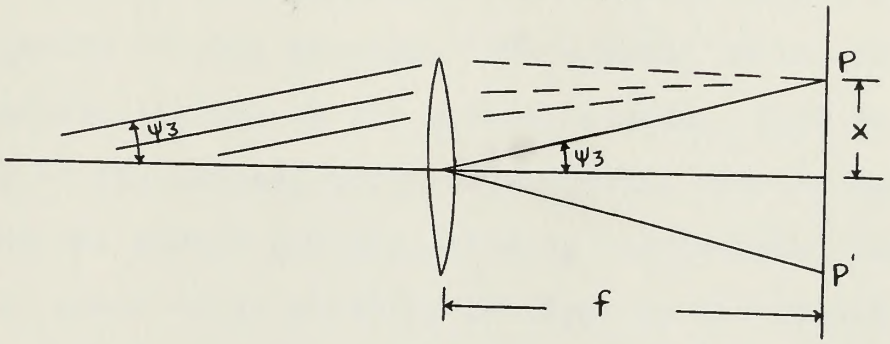


Figure 6

and produce a bright fringe. See figure 6. The distance, x, of the fringe from the center of the fringe pattern is equal to the product of the focal length and the tangent of Ψ_3.

$$x = f \times \tan \Psi_3$$

of Ψ_3, which are parallel, are brought to a common focus by an objective at a point F on the focal plane they will reinforce

POSITION OF FRINGES

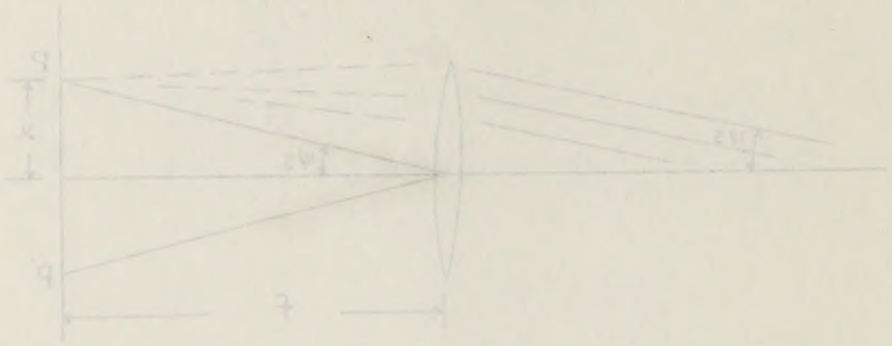


Figure 6

and produce a bright fringe. See figure 6. The distance, x, of the fringe from the center of the fringe pattern is equal to the product of the focal length and the tangent of Ψ_3.

$$x = f \cdot \tan\Psi_3$$

DESCRIPTION OF APPARATUS

CHANGES IN APPARATUS

The apparatus used in this research was essentially the same as that used by Dr. W. H. Robinson[1] in his preliminary investigation of this problem. The changes in the apparatus were centered largely on the optical system. This entailed the removing of the lenses, mirror and special shutter that were between the two Lummer plates, allowing the emergent light from the first plate to be directly incident on the second plate. With less absorption of the light present, the intensity of the spectrum was increased enough so, that the structure of the pattern system obtained could be studied visually more readily and exposure times somewhat reduced. A complete description of the optical system is given on page 39.

RADIATION SOURCE

A discharge tube was connected through a liquid-air trap to a McCleod gauge by means of which the pressures in the tube could be accurately measured. A Hyvac pump also connected to the discharged tube served as a means of controlling and maintaining the pressure at any desired value. A milliammeter in series with the tube served to measure the current. With suitable values of pressure and current it was found that the intensity of the secondary spectrum due to the molecular hydrogen could be made to be relatively weak with respect to the Balmer lines produced by the atomic hydrogen*. Hydrogen was produced by electrolysis and collected in a reservoir which was connected to the tube through a double stopcock; thus admitting small quan-

1 Robinson, W. H., PhD. Thesis, 1937, Boston Univ. Grad.School

* The atomic spectrum forming the regular atomic series may of course be given by atoms which are still components of the molecule. Only at high temperatures would there exist a preponderance of atomic hydrogen.

CHANGES IN APPARATUS

The apparatus used in this research was essentially the same as that used by Dr. W. H. Hoffman[1] in his preliminary investigation of this problem. The changes in the apparatus were centered largely on the optical system. This entailed the removing of the lenses, mirror and special shutter that were between the two Lummer plates, allowing the emergent light from the first plate to be directly incident on the second plate. With less absorption of the light present, the intensity of the spectrum was increased enough so, that the structure of the pattern system obtained could be studied visually more readily and exposure times somewhat reduced. A complete description of the optical system is given on page 34.

RADIATION SOURCE

A discharge tube was connected through a liquid-air trap to a McLeod gauge by means of which the pressures in the tube could be accurately measured. A Hyvac pump also connected to the discharged tube served as a means of controlling and maintaining the pressure at any desired value. A milliammeter in series with the tube served to measure the current. With suitable values of pressure and current it was found that the intensity of the secondary spectrum due to the molecular hydrogen could be made to be relatively weak with respect to the Balmer lines produced by the atomic hydrogen. Hydrogen was produced by electrolysis and collected in a reservoir which was connected to the tube through a double stopcock, thus admitting small quan-

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[1] Hoffman, W. H., Thesis, 1927. Boston Univ. Grad. School

tities of the gas to the tube at will.

The source of radiation was a liquid-air cooled Wood[2] tube of pyrex glass 36cm long and 0.8 cm internal diameter. A wooden pine box coated with glyptol served as a trough for the liquid-air.

The source of excitation was a 30,000 volt transformer with a variable resistance in series with the primary.

A sketch of the apparatus used is given in Fig. 7.

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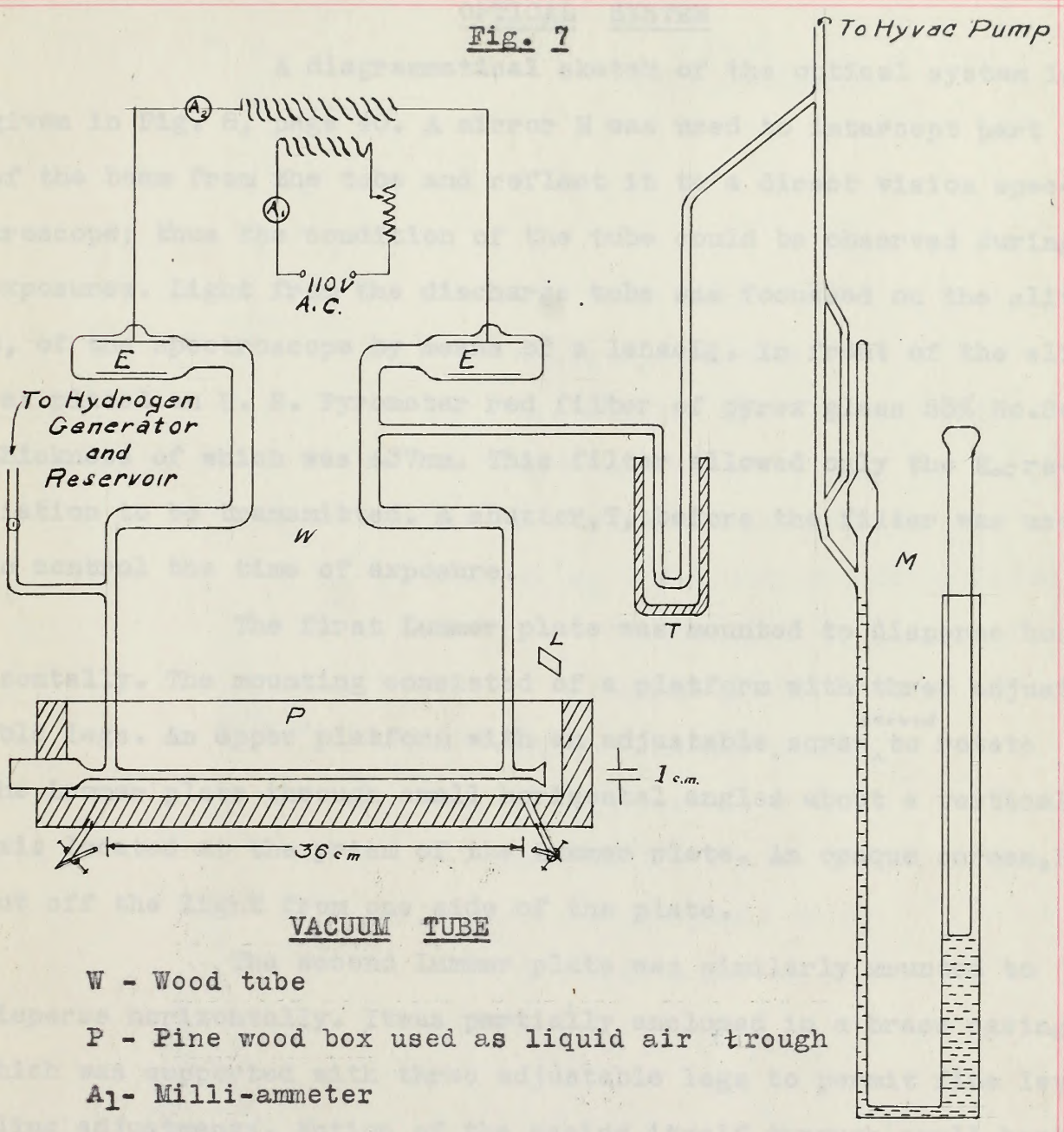
[2] Wood, R. W. , Proc. Roy. Soc. London, A 97, 455 (1920)

Fig. 7



VACUUM TUBE

W - Wood tube

P - Pine wood box used as liquid air trough

A_1- Milli-ammeter

A_2- Ammeter

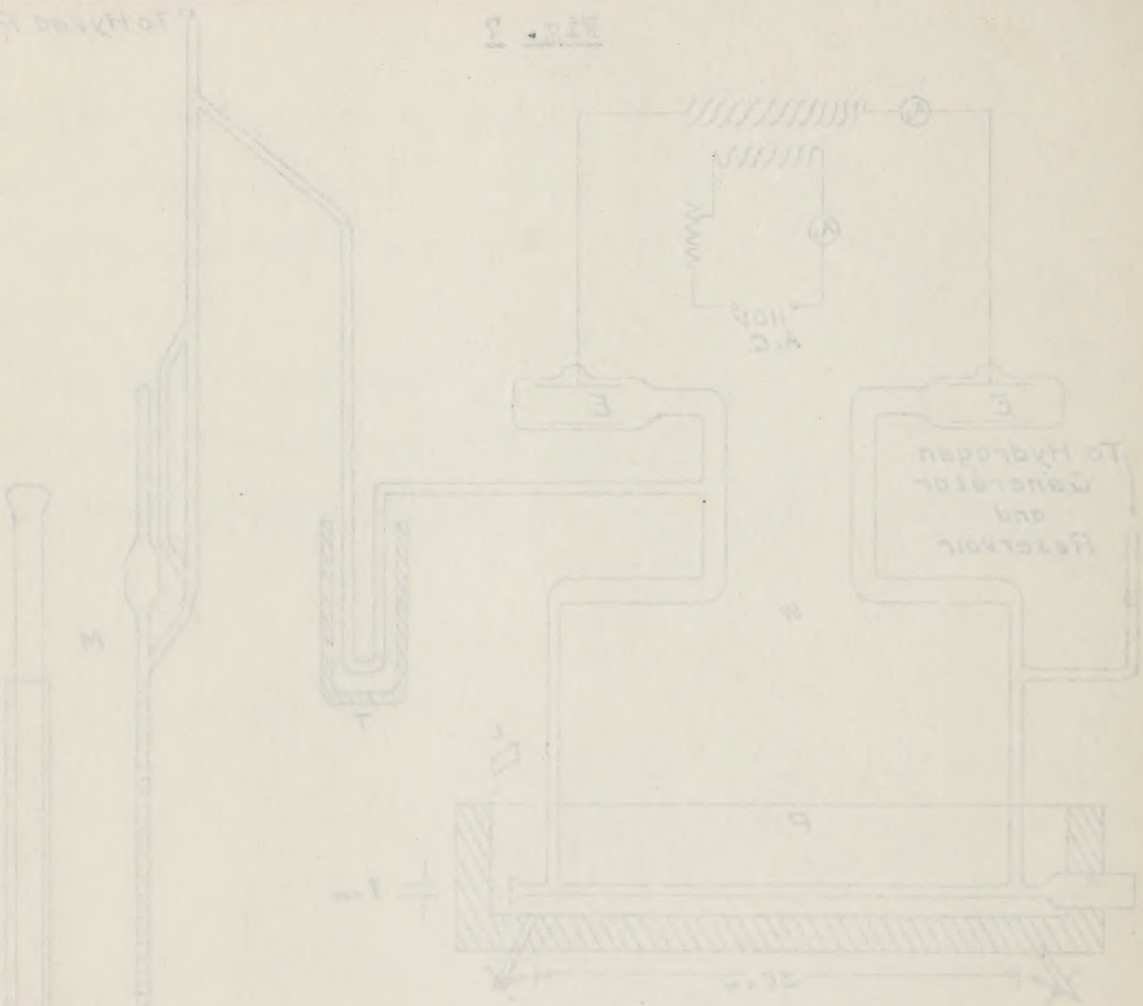
B - High voltage transformer

E - Aluminum electrodes

L - Mirror for viewing height of liquid air around capillary

T - Liquid-air cooled Hg trap

M - McLeod Gauge



OPTICAL SYSTEM

A diagrammatical sketch of the optical system is given in Fig. 8, page 40. A mirror M was used to intercept part of the beam from the tube and reflect it to a direct vision spectroscope; thus the condition of the tube could be observed during exposures. Light from the discharge tube was focussed on the slit S, of the spectroscope by means of a lenseL1. In front of the slit was placed an H. R. Pyrometer red filter of pyrex glass 53% No.241 thickness of which was 4.37mm. This filter allowed only the H_{α} radiation to be transmitted. A shutter,T, before the filter was used to control the time of exposure.

The first Lummer plate was mounted to disperse horizontally. The mounting consisted of a platform with three adjustable legs. An upper platform with an adjustable screw served to rotate the Lummer plate through small horizontal angles about a vertical axis located at the prism of the Lummer plate. An opaque screen,B, cut off the light from one side of the plate.

The second Lummer plate was similarly mounted to disperse horizontally. Itwas partially enclosed in a brass casing which was supported with three adjustable legs to permit fine leveling adjustments. Motion of the casing itself through small horizontal angles was controlled by four set screws located near the corners of the casing.

The telescope and the second Lummer plate are set on a common platform which can be rotated horizontally about a point between the two plates.Thus rotating this platform changes the angle of incidence of the emergent rays of the light from the first plate upon the prism of the second plate.

OPTICAL SYSTEM

A diagrammatical sketch of the optical system is given in Fig. 8, page 40. A mirror M was used to intercept part of the beam from the tube and reflect it to a direct vision spectroscope; thus the condition of the tube could be observed during exposures. Light from the discharge tube was focussed on the slit S, of the spectroscope by means of a lens L. In front of the slit was placed an H. R. Pyrometer red filter of pyrex glass 55% No. 2241 thickness of which was .27mm. This filter allowed only the H$_\alpha$ radiation to be transmitted. A shutter, T, before the filter was used to control the time of exposure.

The first Lumer plate was mounted to disperse horizontally. The mounting consisted of a platform with three adjustable legs. An upper platform with an adjustable screw, to rotate the Lumer plate through small horizontal angles about a vertical axis located at the prism of the Lumer plate. An opaque screen, C, cut off the light from one side of the plate.

The second Lumer plate was similarly mounted to disperse horizontally. These parts enclosed in a brass casing which was supported with three adjustable legs to permit fine leveling adjustments. Motion of the casing itself through small horizontal angles was controlled by four set screws located near the corners of the casing.

The telescope and the second Lumer plate are set on a common platform which can be rotated horizontally about a point between the two plates. Thus rotating this platform changes the angle of incidence of the emergent rays of the light from the first plate upon the prism of the second plate.

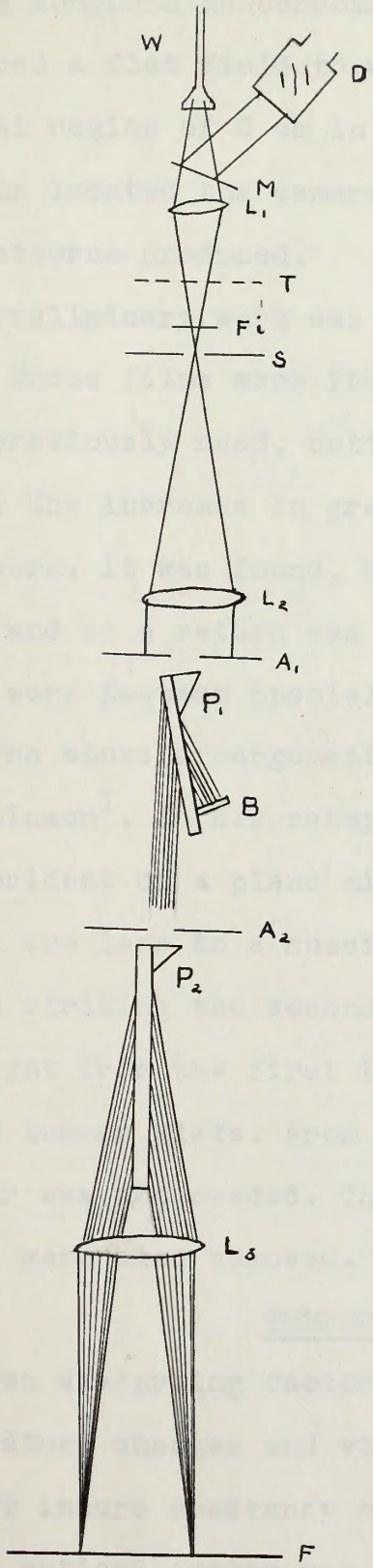


Fig. 8
OPTICAL SYSTEM

W - Wood tube

L_1, L_2, L_3 - Lenses

T - Shutter for time exposure

Fi - Red filter

S - Slit of spectroscope

A_1, A_2 - Shutters for controlling illumination on prisms of Lummer plates

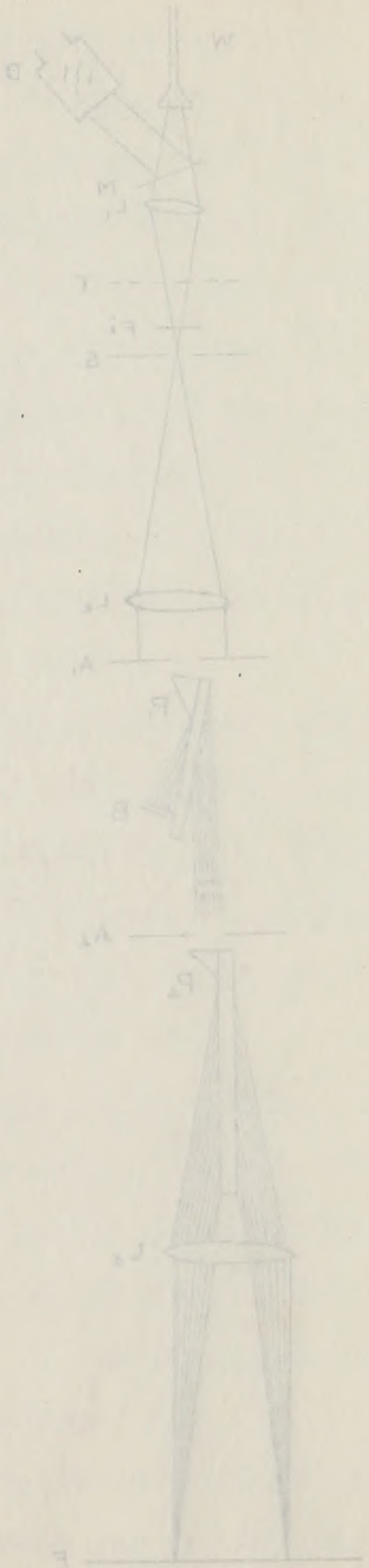
P_1, P_2 - Lummer plates

M - Mirror

D - Direct vision spectroscope

F - Focal plane of L_3

B - Opaque screen



A compound monochromat lens, L_3, made by Bausch and Lomb produced a flat field to within a small fraction of a mm over a central region of 5 cm in diameter. At the focal plane of this lens is located the camera box and an ocular for visual study of the patterns produced.

Preliminary work was done with Agfa films, Emulsion No. 487-534. These films were found to be faster than plates that had been previously used, cutting the time of exposure to at least one fifth. The increase in grain size would not affect the results of this work. It was found, however, that halation entered as a difficulty and so a return was made to backed photographic plates. These were Eastman Special Spectroscopic Plates, Type I-C.

The above arrangement of the apparatus differed from that used by Robinson[1]. In his setup the light coming from the first plate was incident on a plane mirror; the reflected light then passed though one lens to a special shutter and then through a second lens before striking the second Lummer plate. In the present arrangement the light from the first Lummer plate is directly incident upon the second Lummer plate. From Robinson's[2] work it was found that the shutter was not needed. The mirror and the lenses between the Lummer plates were also removed.

TEMPERATURE AND VIBRATION EFFECTS

Two disturbing factors causing shifting of the patterns are temperature changes and vibration.

To insure constancy of temperature of the Lummer plates the entire optical system was housed in a triple walled casing of

---

[1,2] Robinson, W. H., Ph.D. Thesis, (1937) B.U. Grad. School

building paper.The variation in temperature was so small that a further controlled heating circuit within the housing was dispensed with.

The optical system with housing was supported on a concrete table, the concrete and brick legs of which were imbedded in sawdust.

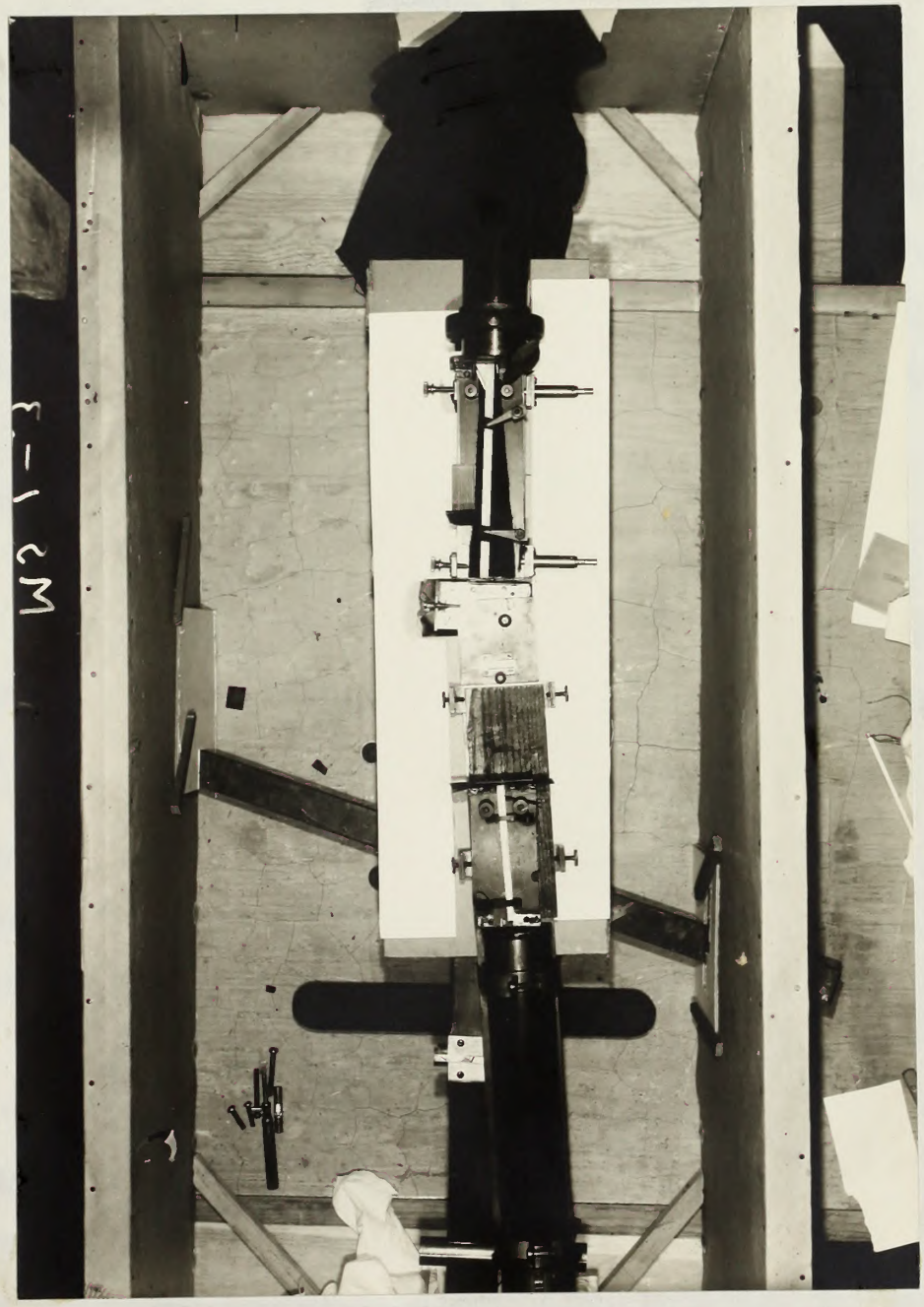
With the above arrangements possible shifts of patterns producing an increase in the spreading of the lines during an exposure were eliminated.

POSITIONS OF LUMMER PLATES

A camera box was set up on a staging above the housing. The top of the housing could easily be removed to expose the Lummer plates, collimator and telescope. The systemwas photographed using Eastman 50,5x7 plates. See photograph on page 43.

To aid in this process strips of white paper were accurately pasted along one edge of the upper surfaces of the Lummer plates and a fine line was also scratched along the axis on the blackened surface of the collimator. Thus a permanent record was obtained of the positions of the plates with respect to the collimator for every set of exposures taken of the patterns.

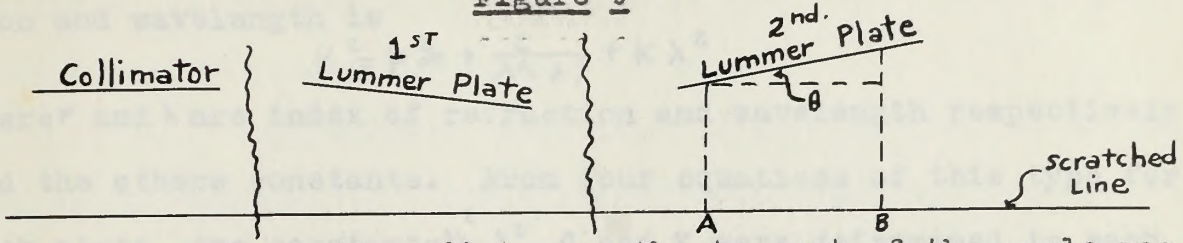
To measure the angle between the collimator and first Lummer plate,and also the angle between the two Lummer plates,a line was scratched with a straight edge on the emulsion side of the photographic plate. See Fig.9. This then was cut into sections containing a Lummer plate image or collimator with a portion of the scratched line. Two points (A,B)were chosen on the line and that part of the photographic plate mounted on a comarator. With the crosshair of the telescope set parallel to the scratched line, distances were then measured from points



OPTICAL SYSTEM WITHIN HOUSING

A and B to line of the Lummer plate edge. Distance AB was also measured. Position of plates and collimator.

Figure 9



From these distances the tangent of the angle was calculated. Similarly the angles for the other sections were determined and from these the angle between the Lummer plates as well as the angle between the first Lummer plate and the collimator were calculated.

CONSTANTS OF THE LUMMER PLATES

The thicknesses of the Lummer plates as given by the makers are known to within one thousandth of a millimeter. See page 48.

The indices of refractions for the various plates were also given for standard wavelengths of light. See table III From these were computed the equations connecting the index of refraction with wavelength for the three plates.

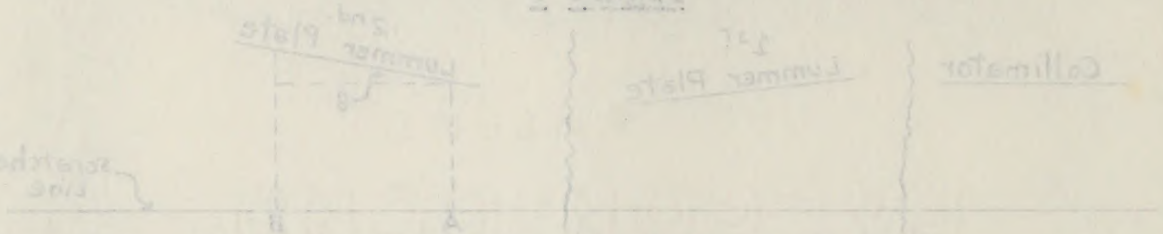
VARIATION OF INDEX OF REFRACTION WITH WAVELENGTH

Table III

Wave Length		Index of Refraction		
		Smallest Pl.	Medium Pl.	Largest Pl.
λ_C	6562.7845 A	1.50746	1.61772	1.57193
λ_D	5892.9485 A	1.50990	1.62261	1.57590
λ_F	4861.327 A	1.51560	1.63492	1.58581
$\lambda_{G'}$	4340.466 A	1.52025	1.64537	1.59417

A and B to line of the Lummer plate edge. Distance AB was also measured. Position of plates and collimator.

FIGURE 2



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VARIATION OF INDEX OF REFRACTION WITH WAVELENGTH
TABLE III

Wave Length	Index of Refraction	
	Smallest Pl.	Medium Pl. Largest Pl.
λ_c 6562.7845 A	1.61772	
λ_c 5892.9405 A	1.62281	
λ_c 4861.327 A	1.63409	
λ_c 4340.466 A	1.64537	

The general equation connecting index of refraction and wavelength is

$$\mu^2 = \mu_\infty^2 + \frac{C}{\lambda^2 - \lambda_1^2} + K\lambda^2$$

where μ and λ are index of refraction and wavelength respectively and the others constants. From four equations of this type for each plate, the constants $\mu_\infty^2, \lambda_1^2$, C and K were determined in each case. The resulting equations are as follows:-

Smallest plate $\mu^2 = 2.2571775 + \frac{.009042497}{\lambda^2 - .030272729} - .017004441\,\lambda^2$

Medium plate $\mu^2 = 2.5620604 + \frac{.023311132}{\lambda^2 - .02932727} - .007245746\,\lambda^2$

Largest plate $\mu^2 = 2.4343834 + \frac{.0171249}{\lambda^2 - .032560549} - .015370676\,\lambda^2$

There remains to be found the angles of the attached prisms of the plates.

To achieve this end the plates were placed on the graduated turntable of a spectrometer in front of the collimator. The slit of the collimator was illuminated by focussing the light of a lamp at a right angle prism situated in front of the lower half of the slit. See Fig. 10. With the correct adjustment of the collimator and the level of the turntable the reflected images of the slits from two sides of the Lummer plate prism could be brought successively into the plane of the slit just above the right angle prism.

For aid in the settings a needle was placed in the plane of the slit; thus the needle and its image in the plane of the slit could be viewed with an eye piece and settings made more accurately. The angle through which the plate was turned for settings of the reflected image of the needle from two sides of the prism gives thus a measure of the angle of the prism.

The angles as determined in this manner are accurate to within ∓ 7.5 seconds and are listed on page 48.

Experiments were made to test whether or not the indices of refraction of the attached prisms are the same as that for the plates. It was found that they were the same within experimental error for the smallest and medium plate. For the prism of the largest plate μ for H_α = 1.57531.

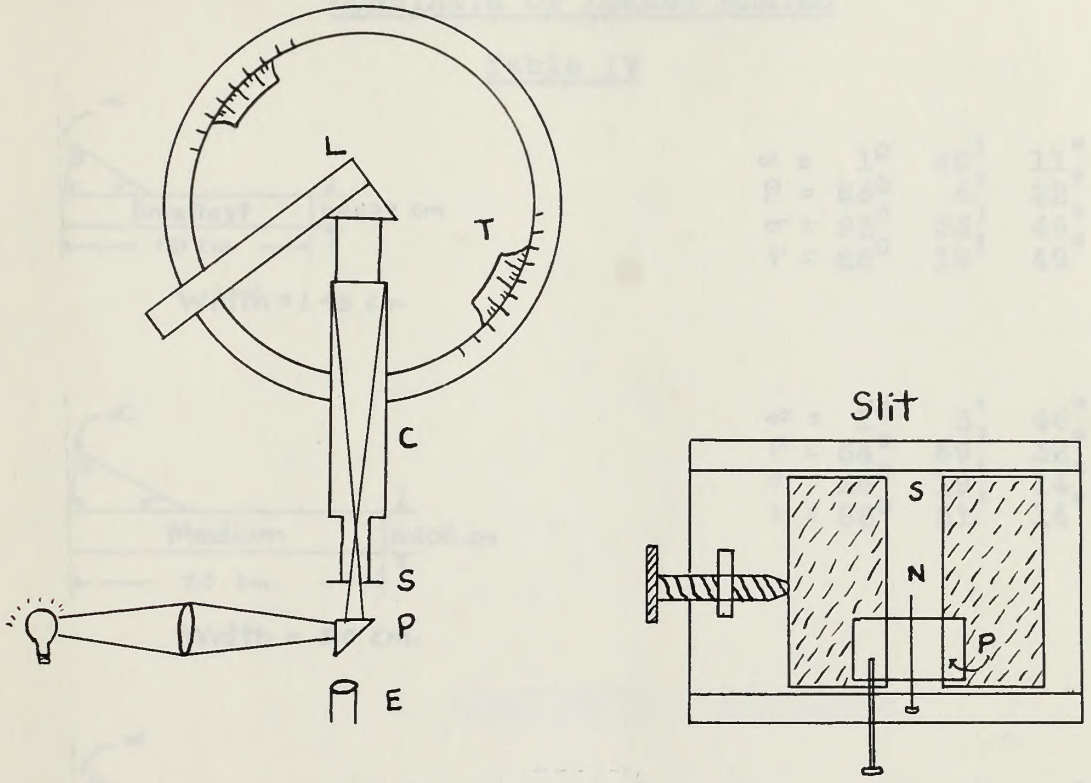


Fig. 10

OPTICAL ARRANGEMENT FOR MEASURING THE ANGLES OF LUMMER PLATE PRISM

T - Spectrometer turntable

C - Collimator

L - Lummer plate

S - Slit

P - Right angle prism

N - Needle

E - Eye piece

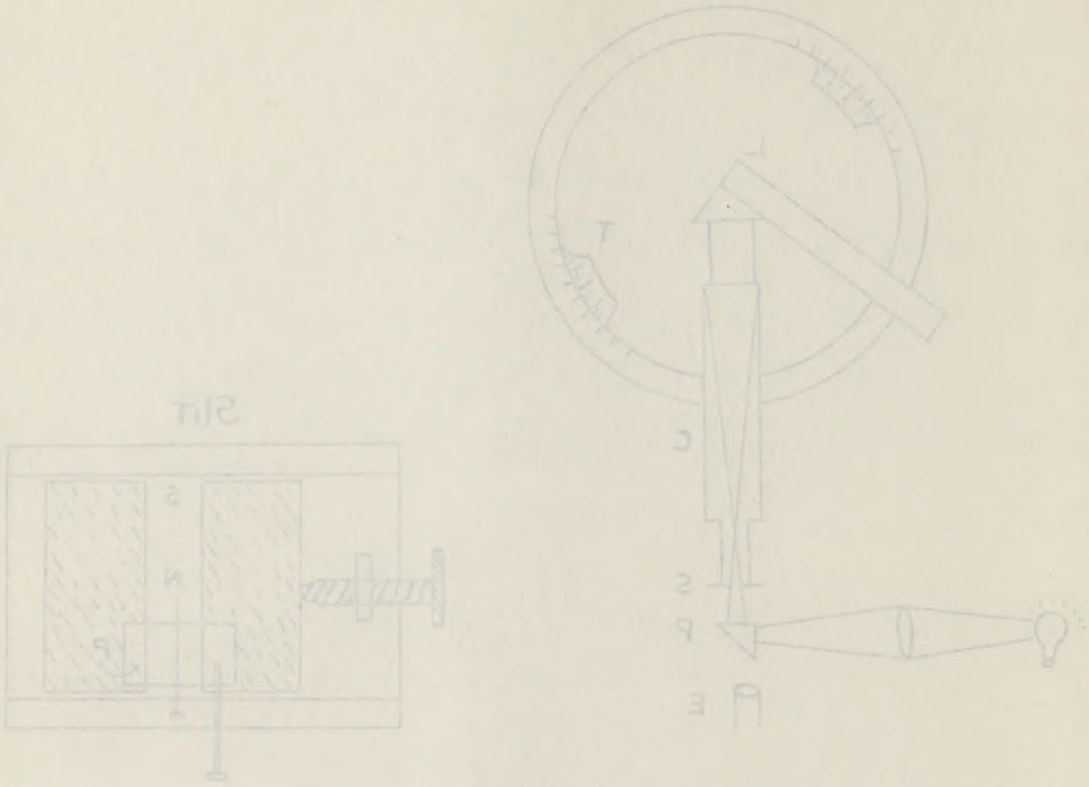


Fig. 10

OPTICAL ARRANGEMENT FOR MEASURING THE ANGLE
OF
LUMMER PLATE PRISM

T - Spectrometer turntable

C - Collimator

L - Lummer plate

S - Slit

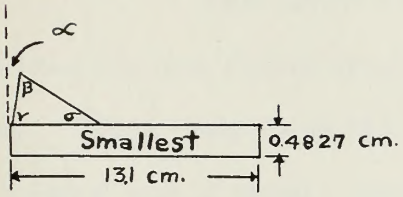
P - Right angle prism

N - Nealle

E - Eye piece

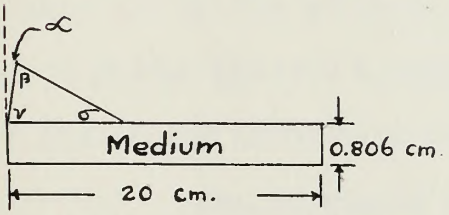
CONSTANTS OF LUMMER PLATES

Table IV



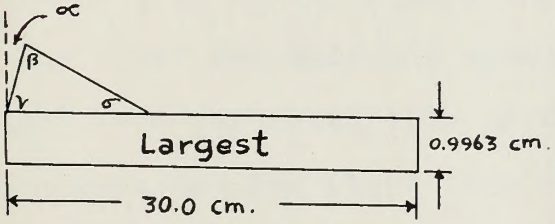
Width = 1.45 cm.

$\alpha = 1^{\circ}\ 40'\ 11''$
$\beta = 66^{\circ}\ 6'\ 22''$
$\sigma = 25^{\circ}\ 33'\ 49''$
$\gamma = 88^{\circ}\ 19'\ 49''$



Width = 3.0 cm.

$\alpha = 1^{\circ}\ 8'\ 46''$
$\beta = 64^{\circ}\ 49'\ 32''$
$\sigma = 26^{\circ}\ 19'\ 14''$
$\gamma = 88^{\circ}\ 51'\ 14''$



Width = 3.98 cm.

$\alpha = 1^{\circ}\ 28'\ 25''$
$\beta = 65^{\circ}\ 19'\ 3''$
$\sigma = 26^{\circ}\ 9'\ 22''$
$\gamma = 88^{\circ}\ 31'\ 35''$

$\mu_{H\alpha}$ for smallest plate and prism = 1.50746
$\mu_{H\alpha}$ for medium plate and prism = 1.61772
$\mu_{H\alpha}$ for largest plate - - - - - - - = 1.57187

} Hilger Values

$\mu_{H\alpha}$ for attached prism of largest plate - - - - - - - - - - = $1.5753 \mp .001$

EXPERIMENTAL PROCEDURE

OPTICAL ADJUSTMENTS

The care of the Lummer plates is extremely important--the surfaces must be very clean and especially free from grease. This was insured by rubbing the surfaces gently with lens paper and a small amount of alcohol. The Lummer plates were then set in their holders and shutters were adjusted so that light would only enter the attached prisms. Great care was exercised in clamping the plate in place for any appreciable local pressure on the plate may seriously impair its definition.

The complete optical system was as shown in Fig. 8, page 40. The first Lummer plate was rotated through small horizontal angles by means of an adjustable screw on the platform until the light was emergent somewhat near the grazing angle. The light from the prism side of the plate was cut off by an opague screen and the light from the other side allowed to fall on the second plate. The intensity of the light emergent from the free side was found to be greater than that from the prism side, apparently due to the one more reflection and corresponding loss of light of the first refracted beam and absorption through glass before finally emerging on the prism side of the plate. The second Lummer plate was set so that its prism was on the opposite side to that of the first prism of the first Lummer plate. This in part, thus corrects for slight curvature of the lines inherent in the prism. With light now filling the prism of second Lummer plate, the platform supporting both the telescope and second Lummer plate was rotated through small horizontal angles until a desired pattern system was obtained.

Final adjustment in the leveling of the second plate was then made so that the maximum and minimum of the group pattern were parallel to the individual lines. The second plate itself was then rotated until the desired number of groups on either side of the center of the fringe system was obtained. In general the second Lummer plate was set so that more groups were obtained from the plane side of the plate since this was the side of greater intensity. In the process of fixing the position of the second Lummer plate, slight readjustment had to be made in the position of the first plate so that the aperature of the second plate was completely filled with light.

The patterns were studied visually at all times by one observer while another made the adjustments. Any slight changes in either of the two plates themselves, necessitated the rotating of the platform which supported as a whole the telescope, camera box, and second Lummer plate for best conditions of intensity and structure. After a particular pattern was decided upon during this long and tedious adjustment and readjustment, the telescope and plateholder were clamped in position, and the housing set over the complete optical system.

DISCHARGE CONDITIONS

Before an exposure the mercury trap as well as the pine wood box enclosing the capillary of the discharge tube was filled with liquid air. The tube was then run for about five minutes with the vacuum pump continually exhausting, the tube

itself being flushed several times with fresh hydrogen from the reservoir. This procedure removed thoroughly any mercury vapor and set the tube for steady conditions during an exposure. The spectrum was continually observed by a constant deviation spectroscope. With the secondary spectrum almost absent the desired pressure and secondary current were obtained and kept constant for any one exposure. The pressures and the current densities for the different plates were kept approximately 1.0 mm. of Hg. and 22 ma/cm^2 respectively. See table V, page 55. In all cases the red filter was used, allowing only the light giving H_α to be transmitted.

PHOTOGRAPHIC PROCEDURE

In general for each setting of the Lummer plates three photographic plates were taken with exposures from 15 to 40 minutes. At the beginning of this research, films which were relatively fast were used; thus saving in exposure time and cost of liquid air. Because of halation, however, a shift was made to Eastman Special Spectroscopic Plates, Type I-C which were properly backed. These plates proved to be not very satisfactory for they were not very clear apparently subject to chemical fog. At a later date Eastman plates, Type 144-F were tried. It was unfortunate that these were not used at the very beginning since they produced very clear spectrograms. All plates were developed by Eastman's D-19 formula, with development time about six minutes.

After the exposure of three photographic plates

for different times for a given pattern the top of the housing was removed and a photograph taken from overhead of the alignment of the collimator, telescope and Lummer plates. Eastman 50, 5 x 7 plates were used here.

SPECTROGRAMS AND DATA

TABLE V

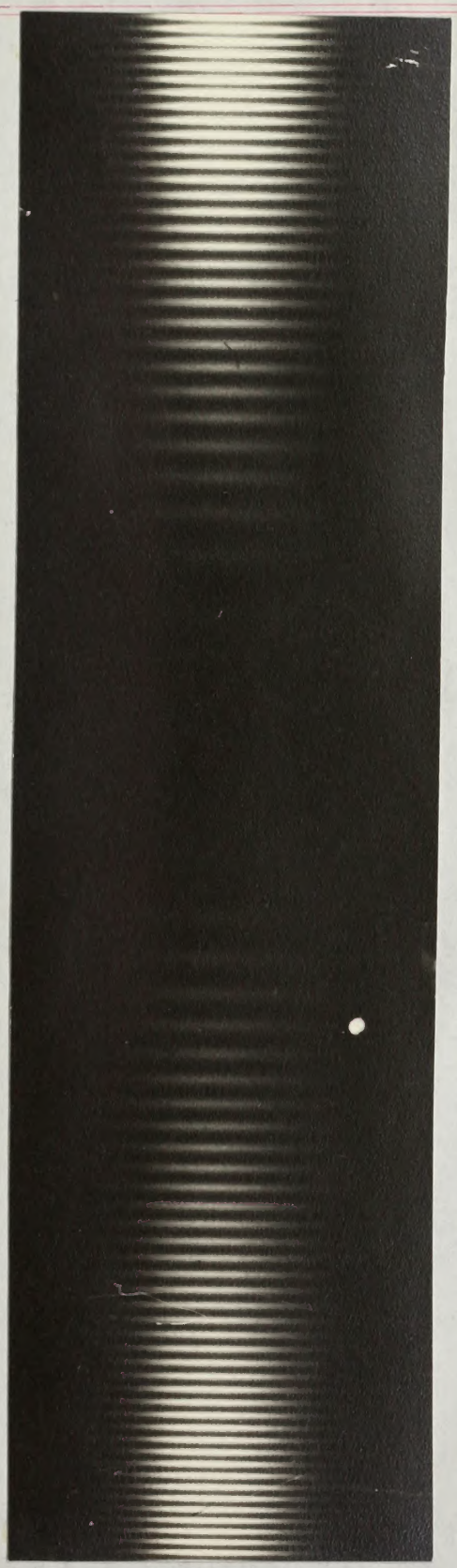
Plate No.	Vacuum (mm of Hg)	Current ma/cm^2	Exposure
L	1.80	18.5	5 sec.
M	0.92	22.0	15 sec.
S	0.92	22.0	30 sec.
SL-9	0.93	22.0	27 min.
SL-2	1.80	20.5	22 min.
ML-3 (film)	1.70	18.0	25 min.
SM-14 (Film)	2.50	27.0	3 min.
SM-9	1.50	32.0	1.5 min.
MS-3	0.80	21.0	45 min.

TABULATION OF PLATES

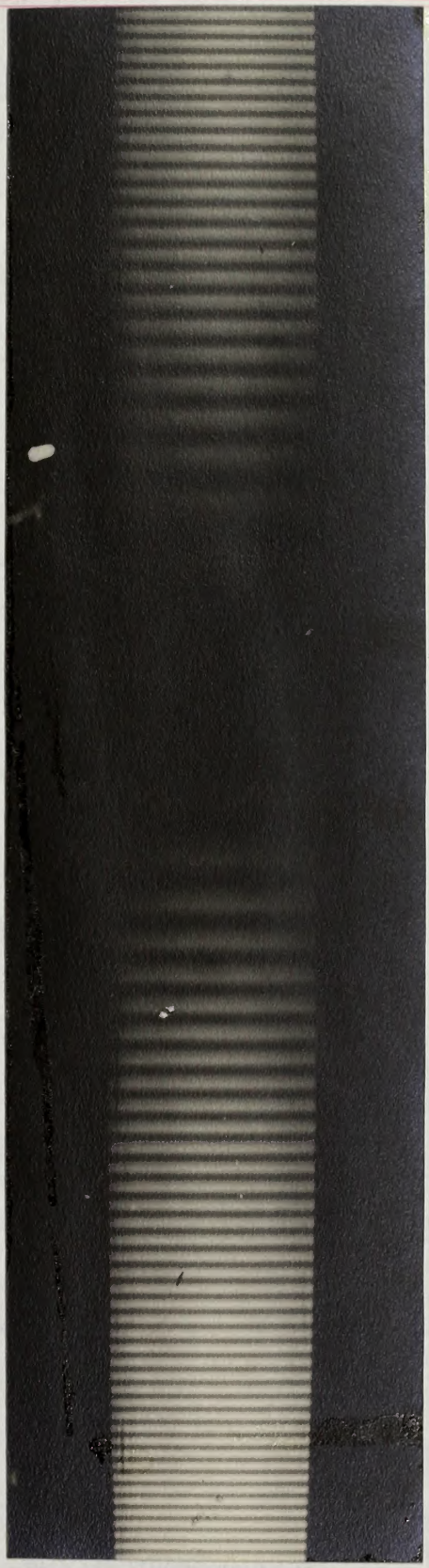
TABLE V

Plate No.	Vacuum (mm of Hg)	Current ma/cm^2	Exposure
L	1.80	18.8	3 sec.
M	0.95	22.0	15 sec.
S	0.92	24.0	30 sec.
S1-2	0.92	26.0	47 min.
S1-3	1.80	26.0	22 min.
S2-3	1.70	18.0	25 min.
S2-1A (filter)	3.80	17.0	3 min.
S2-3 (filter)	1.80	23.0	1.6 min.
S2-3	0.80	17.0	45 min.

TABULATION OF PLATES

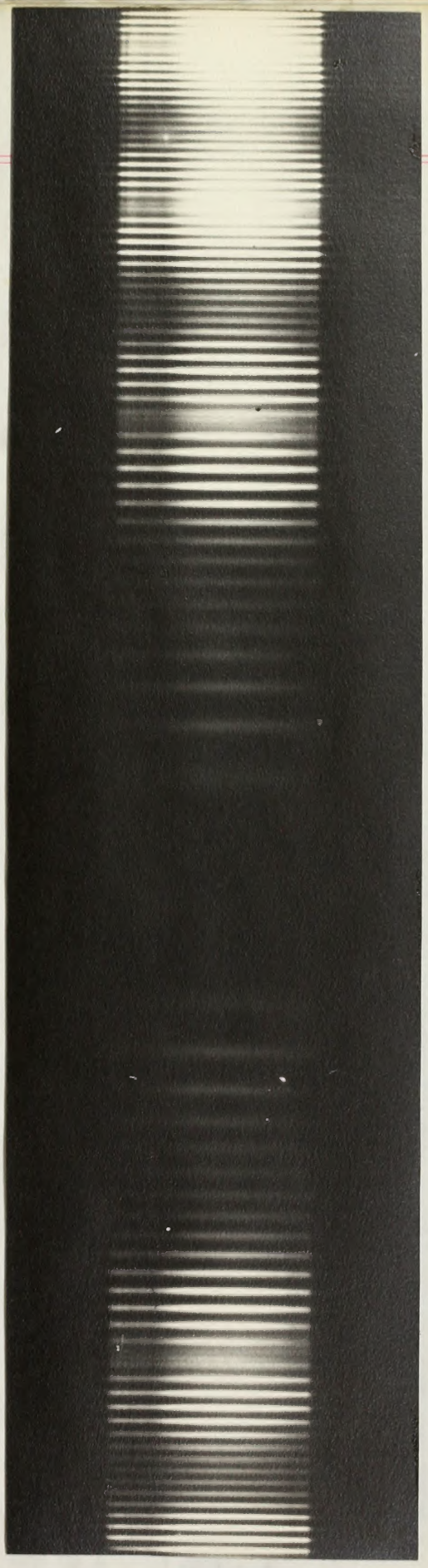


L

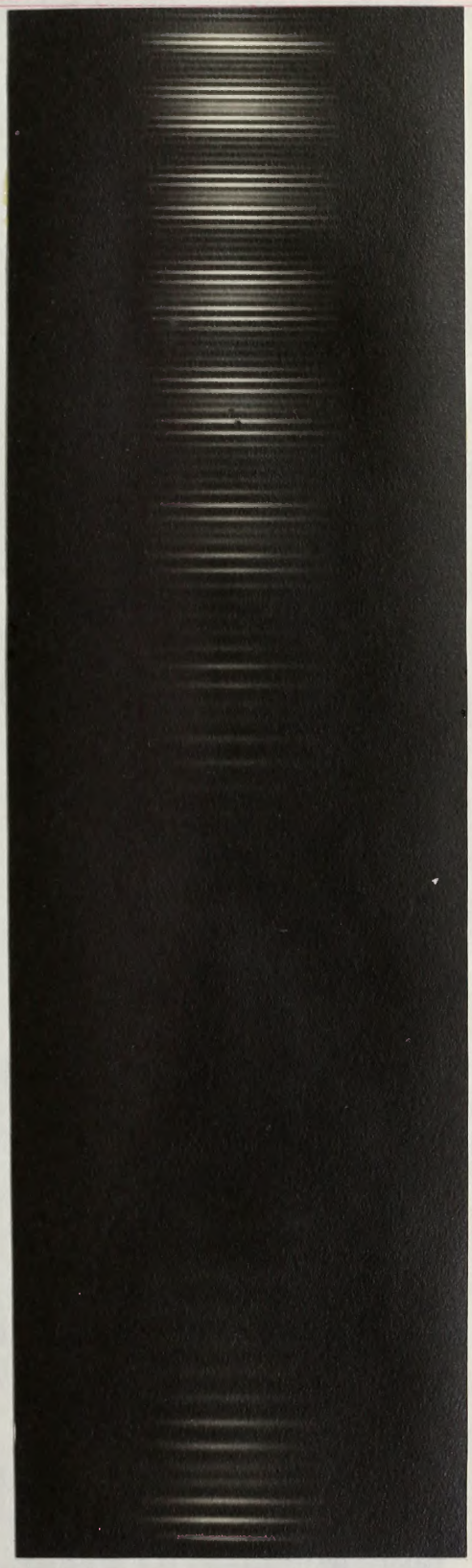


M

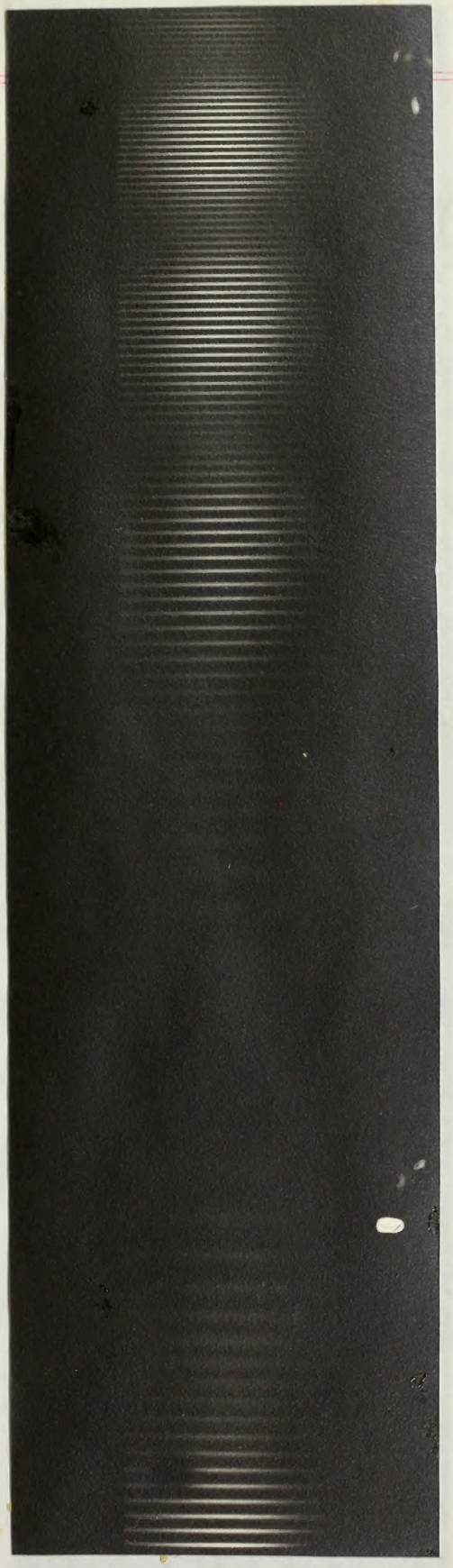
S



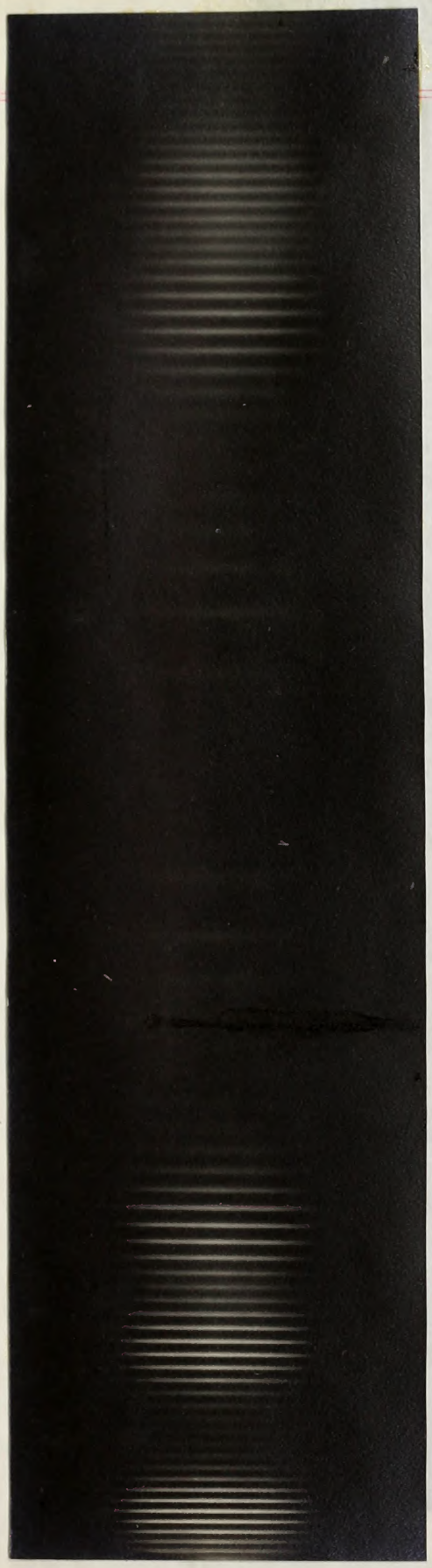
SL-9



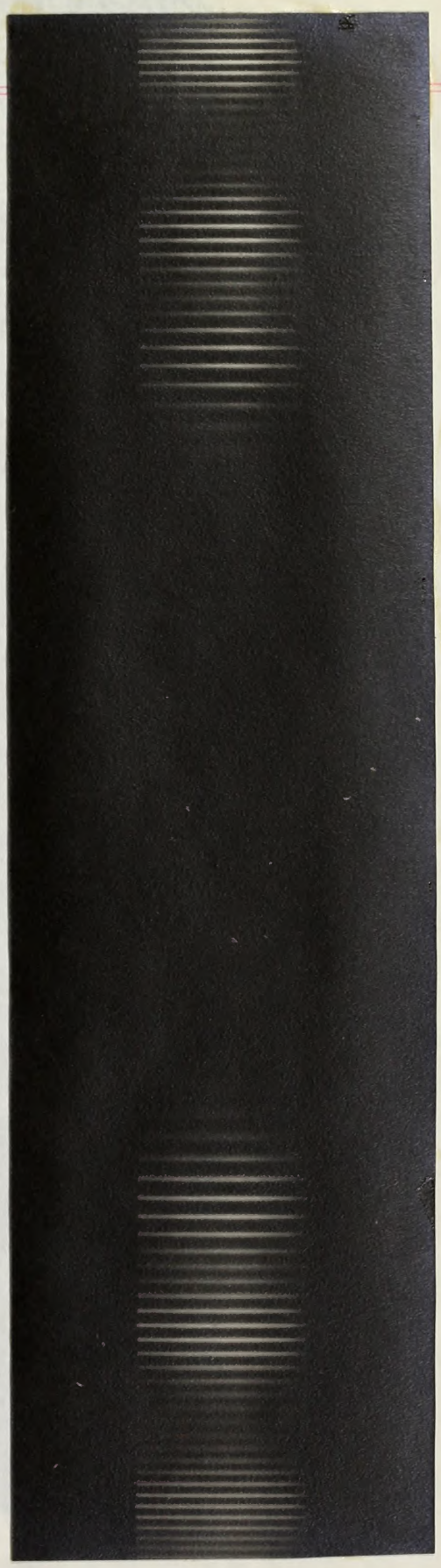
SL-2



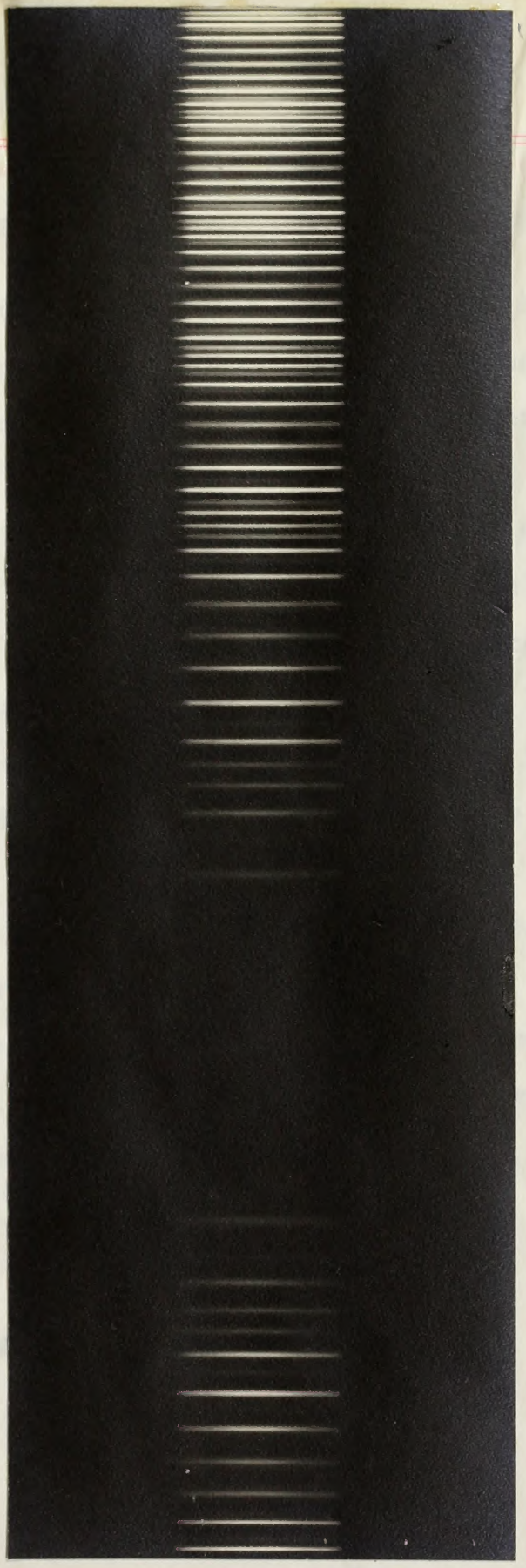
ML-3



SM-14



SM-9



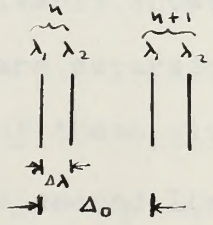
MS-3

DISCUSSION OF SINGLE PLATE SPECTROGRAMS

A list of the plates and films that were obtained under different conditions are given in table V showing the pressure, current and exposure time. The letters L, M and S used to distinguish the various spectrograms stand for the longest, medium and smallest Lummer plate respectively as described in the section "Constants of the Lummer plates". Where two letters appear, the photographic plate was obtained with two Lummer plates in tandem with the order of the letters L, M and S giving the order of the Lummer plates. Enlarged photographs of the original photographic plates are shown on pages 56-60.

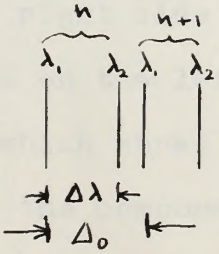
The first three photographs L, M and S are the single Lummer plate patterns. The separation of the two main components of H_α and the separation of the successive orders are related in the following way for the 3 Lummer plates.

Δ_o = order separation
$\Delta\lambda$ = main component separation



$\Delta\lambda = \frac{1}{3}\Delta_o$

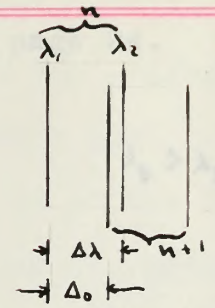
Smallest Plate



$\Delta\lambda = \frac{2}{3}\Delta_o$

Medium Plate

Largest Plate



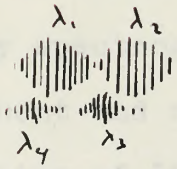
$\Delta\lambda \sim 1.2\,\Delta_0$

In plate L we see the lines appear very broad due to the fact that $\Delta\lambda \sim 1.2\,\Delta_0$, i.e., the component of one order overlaps the main component of the next order. The photograph obviously shows that the largest plate cannot be used alone for fine structure work. Passing to plates M and S we find no overlaping of the orders. The components are just separated by plates M and to a larger degree larger in plate S. The smallest Lummer plate is the best and gives the best definition.

DISCUSSION OF DOUBLE PLATE SPECTROGRAMS

The remaining photographs are those taken with combinations of two Lummer plates in tandem. Using the smallest Lummer plate first and then the largest in tandem, plates SL-9 and SL-2 were obtained. SL-9 clearly shows the group pattern. Both main components λ_1 and λ_2 are separated in the first smallest Lummer plate and each of these gives rise to a distinct series of lines due to the second Lummer plate. There are almost 3 full groups on the right side of the center of the pattern system and $1\frac{1}{2}$ groups on the left. There is present some intergroup structure which shows the existence of a third component. Theoretically the components would be arranged in the following manner with λ_3 to the left of λ_2

and λ_4 to the left of λ_1 . See page 24.



$\lambda_4 > \lambda_2$

Plate SL-2 was taken with the light emergent from the first plate at a greater angle; thus with higher orders the components were closer together and many groups were obtained with but few lines representing each individual component in the final pattern. This plate failed to show any of the minor components, due apparently to loss in resolving power from failure in using the entire length of the first Lummer plate as well as to using higher orders. It is evident then in seeking the fine structure, light must emerge from the first Lummer plate at nearly grazing angle; thus utilizing the full length of the plate. However, it was found that there was an optimum angle for good intensity just beyond grazing emergence. The effective number of reflections at this angle is approximately two-thirds of the maximum possible; thus the resolving power is two-thirds of the theoretical value, or very closely $\frac{2}{3}\frac{\ell}{\lambda}(\mu^2-1)$. The use of the lower orders of the first plate would give greater dispersion of the lines in the final pattern and so aid the study of the fine structure.

The following plate ML-3 where the medium Lummer plate was used as the auxiliary spectrograph gave but slight evidence of the third component and none of the fourth. There appears no distinct grouping of lines for main components 1 and 2 as in the other combination of plates.

Films SM-14 and SM-9 gave distinct main component groups. The intergroup structure which was present showed the existence of a third component.

By far, the most interesting of the plates obtained was that obtained with a combination of the medium and smallest Lummer plates where the medium plate was used as the auxiliary spectrograph. The pattern system obtained with this combination is shown in plate MS-3. From direct measurements of the lines with a comparator and a study of the microphotometer tracing made of the plate, the lines fell into four distinct series when the intensities of the lines were plotted against their relative positions on the plate. This then gives complete evidence of the third and fourth components. A detailed discussion of this plate will be given in the next section.

RESULTS

THEORETICAL CALCULATIONS OF THE STRUCTURE OF PLATE MS-3

The diagram below gives the positions of the two Lummer plates for photographic plate M-S 3.

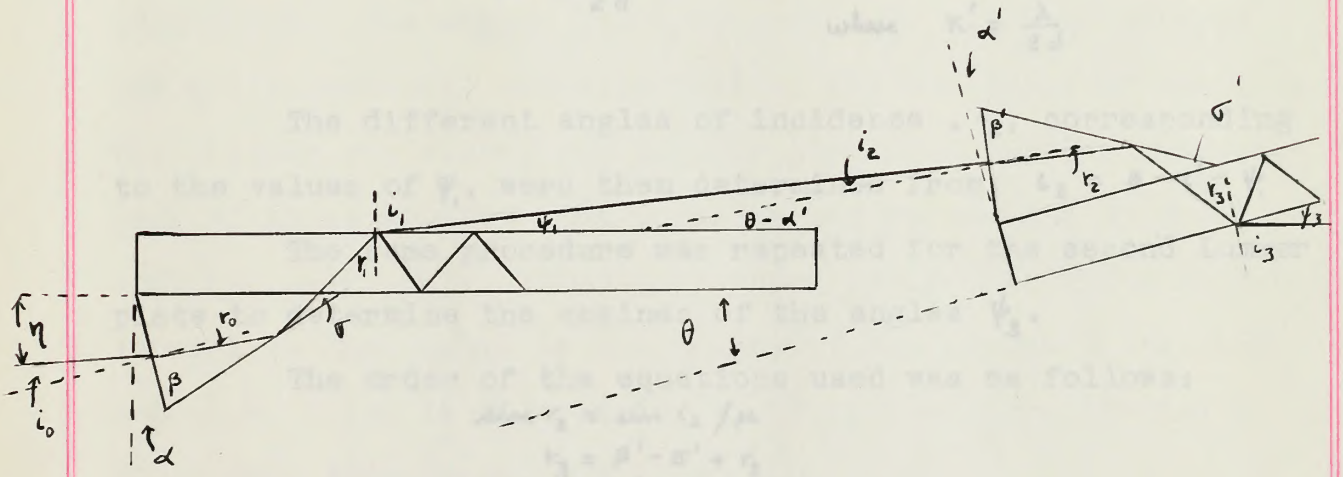
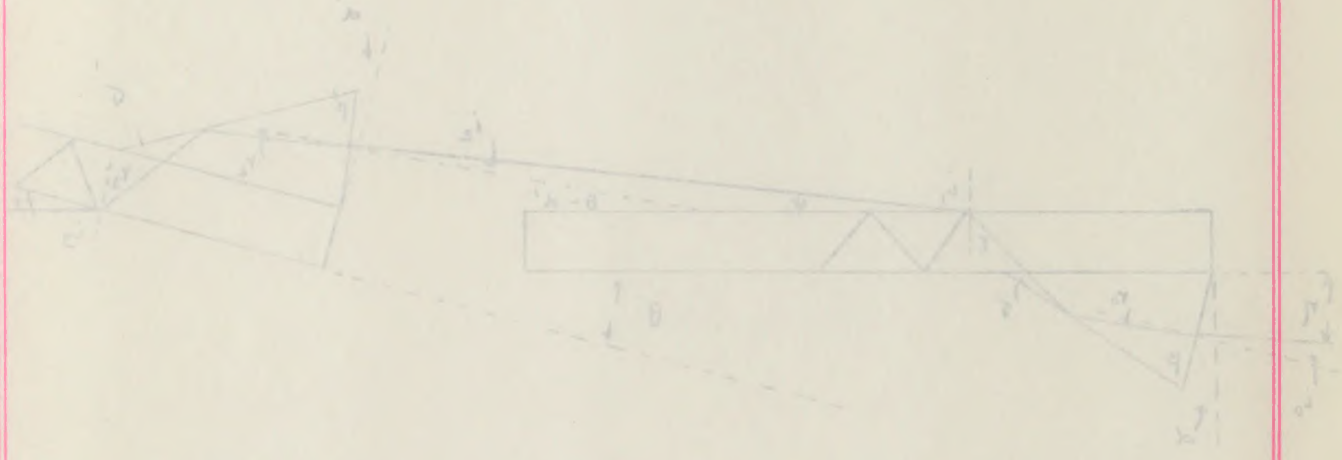


Fig. 11

POSITIONS OF LUMMER PLATES FOR MS-3

The angles θ and η were determined by the method described in the section "Positions of Lummer Plates" page 42. The angle of incidence , i_0, was then found from the equation $i_0 = \alpha - \eta$, and also angle r_0 from the relation $\sin r_0 = \sin i_0 / \mu$. Angle r_1 followed from the equation $r_1 = \beta - \sigma - r_0$. The cosine of ψ_1 was then determined from $\cos \psi_1 = \sin i_1 = \mu \sin r_1$

With the above values of ψ_1, and r_1 , the integral order number n was calculated from the equation $n_M = \frac{2d \cos \psi}{\lambda \tan r_1}$.



Following this, values of the cosines of the r'_2 and ψ'_4 for values of n above and below n_M from the two equations

$$\cos r_1 = \frac{\lambda}{2d\mu} n = K n \qquad \text{where } K = \frac{\lambda}{2d\mu}$$

$$\cos \psi_1 = \frac{\lambda}{2d} n \tan r_1 = K' n \tan r_1 \qquad \text{where } K' = \frac{\lambda}{2d}$$

The different angles of incidence , i_2, corresponding to the values of ψ_1, were then determined from $i_2 = \theta - \alpha' - \psi_1$

The same procedure was repeated for the second Lummer plate to determine the cosines of the angles ψ_3.

The order of the equations used was as follows:

$$\sin r_2 = \sin i_2 / \mu$$

$$r_3 = \beta' - \sigma' + r_2$$

$$\cos \psi_3 = \mu' \sin r_3$$

$$n'_M = \frac{2d' \cos \psi_3}{\lambda \tan r_3}$$

$$\cos r_3 = \frac{\lambda}{2d'\mu'} n' = L n' \qquad \text{where } L = \frac{\lambda}{2d'\mu'}$$

$$\cos \psi_3 = \frac{\lambda}{2d'} n' \tan r_3 = L' n' \tan r_3 \qquad \text{where } L' = \frac{\lambda}{2d'}$$

The distances , X , of the lines as measured from the center of the pattern system on the microphotometer curve were then calculated from the equation $X = 50 F \tan \psi_3$. F is the focal length of the telescope lens . Since the scale of the microphotometer tracing was 50:1 the values of x had to be multiplied by 50.

All the equations as mentioned above are fully developed and discussed in the section "Theory of two Lummer

plates in tandem", page 31.

The constants used as a basis for the above calculations are listed in table VI .

In this table are listed the comparative values of the index of refraction for the theoretical wavelengths of the five components. The angles θ and i_0 as used were slightly altered for the theoretical calculations of the positions of the lines. The changes are within the experimental error of the measurement of these angles. The experimental values are $\theta = 6^\circ 34' 55''$ and $i_0 = 32' 57''$. It will be shown later that these angles as well as the angles of the prisms of the Lummer plates are not as critical in determining the final patterns as they at first were thought to be.

The results of the calculations are listed in the following tables.

TABLE VI

MEDIUM LUMMER PLATE

Comp.	λ (A°)	μ	$2d$ (cm)	$\beta - \sigma$
5	6562.8978	1.6177193		
4	6562.8513	1.6177196		
1	6562.8358	1.6177197	1.612	38° 30' 18"
3	6562.7406	1.6177203		
2	6562.6941	1.6177205		

SMALLEST LUMMER PLATE

Comp.	λ A°	μ'	$2d'$ (cm)	$\beta' - \sigma'$
5	6562.8978	1.5074597		
4	6562.8513	1.5074598		
1	6562.8358	1.5074599	.9654	40° 32' 33"
3	6562.7406	1.5074602		
2	6562.6941	1.5074603		

i_0	θ
0° 39' 0"	6° 31' 54"

i_0 = angle of incidence
θ = angle between plates

MEDIUM PLATE

	n	cos r_1	cos r_1	cos ψ_1	ψ_1
(1)	31268	2.5166523×10^{-5} n	.78690684	.99824916	3° 23' 27.40"
(2)	31268	2.5165968×10^{-5} n	.78688949	.99828543	3° 23' 20.00"
(3)	31268	2.5166149×10^{-5} n	.78689515	.99827361	3° 22' 2.03"
(4)	31268	2.5166584×10^{-5} n	.78690875	.99824514	3° 23' 41.59"
	31267	"	.78688358	.99829713	3° 20' 39.18"

TABLE VII

CALCULATIONS OF ψ_1's

MEDIUM PLATE

	n	cos r_1	cos ψ	ψ_1
(1)	31268	$2.81663253 \times 10^{-5}$ n	.78699664	.99834916	3° 53' 27.60"
(2)	31268	$2.81659968 \times 10^{-5}$ n	.78688949	.99838545	3° 23' 70.00"
(3)	31268	$2.81661469 \times 10^{-5}$ n	.78689615	.99837581	3° 22' 8.05"
(4)	31268	$2.81665844 \times 10^{-5}$ n	.78690375	.99834514	3° 23' 41.39"
	31267	"	.78688538	.99839713	3° 20' 39.18"

TABLE IX

CALCULATIONS OF Ψ_1's

TABLE VIII

CALCULATIONS OF ψ_3's

Comp. No	cos r_3	r_3	n'	cos r_3	cos ψ_3	tan ψ_3
(1) 1° 28' 16.4"	cos r_3 = 4.5096046 x 10^{-5} n		16598	.74850417	.99964180	.026771830
			16599	.74854927	.99956539	.o29492480
			16600	.74859436	.99948843	.03199860
			16601	.74863946	.99941177	.03431496
			2	.74868456	.99933500	.03648767
			3	.74872965	.99925824	.03853776
			4	.74877475	.99918138	.04048758
			5	.74881984	.99910460	.04234708
			6	.74886494	.99902776	.04412911
			7	.74891004	.99895106	.04583905
			8	.74895513	.99887411	.04749235
(2) 1° 30' 23.0"	cos r_3 = 4.5095060 x 10^{-5} n		16593	.74822633		
			4	.74830743	.99997700	.0067875
			5	.74835252	.99990020	.01412979
			6	.74839762	.99982188	.01887874
			7	.74844271	.99974718	.02249125
			16598	.74848781	.99967020	.02568868
			9	.74853290	.99959355	.02852000
			16600	.74857800	.99951663	.03110503
			1	.74862309	.99943984	.03348242
			2	.74866819	.99936303	.03571009
			3	.74871328	.99928654	.03779470
			4	.74875838	.99920945	

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CALCULATIONS CONTINUED

Comp.	i_2	Cos r_3	n	cos r_3	cos Ψ_3	tan Ψ_3
(3) 31268	$1^\circ\ 29'\ 40.97''$	cos r_3=4.5095383 x 10^{-5}n	6	.74840248	.99981421	.01927811
			7	.74844807	.99973771	.02290880
			8	.74849317	.99966098	.02604525
			16599	.74853826	.99958443	.02883950
			16600	.74858336	.99950740	.03140038
			1	.74862845	.99943068	.03375923
			2	.74867355	.99935387	.03596526
			3	.74871864	.99927725	.03863985
			4	.74876374	.99920033	.04001530
(4) 31268	$1^\circ\ 28'\ 1.41''$	cos r_3=4.5096156 x 10^{-5} n	0	.74859619	.99950047	.03162137
			1	.74864129	.99940860	.03440681
			2	.74868638	.99933180	.03657406
			3	.74873148	.99925507	.03862130
			16604	.74877657	.99917825	.04056597
			5	.74882168	.99910139	.04242264
			6	.74886677	.99902460	.04419992
			7	.74891186	.99894783	.04590972
			8	.74895696	.99887093	.04755954
(4) 31267	$1^\circ\ 31'\ 3.82''$	cos r_3 = 4.5096156x10^{-5}n	1	.74819032		
			2	.74823542		
			3	.74828052		
			4	.74832561	.99994570	.0104239
			16595	.74837071	.99986902	.02038925
			6	.74841580	.99979220	.02385295
			7	.74846090	.99971561	.02688342
			8	.74850600	.99963881	.02959754
			9	.74855109	.99956226	
			16600	.74859619	.99950047	

CALCULATIONS CONTINUED

Date	$\frac{1}{r^2}$	$\cos^2\frac{\theta}{2}$	$\cos^3\frac{\theta}{2}$	$\cos\frac{\theta}{2}$	$\tan\frac{\theta}{2}$
31/5/48 (3)	1° 58' 13.5" $\times 10^{-4}$	$\cos^2\frac{\theta}{2}=+.9999808002$			
		6	.7486406246	.9999142431	.0130731213
		7	.7486406207	.9999713771	.0129908860
		8	.7486423717	.9999685096	.0129604253
		16599	.7486538228	.9999884413	.0129839360
		16600	.7486525136	.9999850740	.0131400398
		1	.7486529845	.9999830868	.0132793525
		2	.7486527456	.9999830897	.0132902226
		3	.7486527164	.9999827868	.0132832905
		4	.7486526374	.9999820028	.0132701530
31/5/48 (4)	1° 58' 13.5" $\times 10^{-4}$	$\cos^2\frac{\theta}{2}=+.9999808002$			
		6	.7486396129	.9999300647	.0131623157
		7	.7486841239	.9999680860	.0134406231
		8	.7486866238	.9999733160	.0136574106
		3	.7486721318	.9999585007	.0136623180
		16604	.7486772657	.9999726525	.0140586527
		5	.7486621659	.9999610139	.0142422564
		6	.7486864777	.9999850460	.0144139852
		7	.7486911186	.9999947263	.0145900271
		8	.7486956196	.9999870959	.0147089934
31/5/48 (4)	1° 58' 13.5" $\times 10^{-4}$	$\cos^2\frac{\theta}{2}=+.9999808002$			
		1	.7486190292		
		2	.7486235642		
		3	.7486280592		
		4	.7486325671	.9999948470	.0104239
		16599	.7486370771	.9999869028	.0103859259
		6	.7486412850	.9999793220	.0103858265
		7	.7486456550	.9999713611	.0103888242
		8	.7486500600	.9999633881	.0103859784
		9	.7486545109	.9999553288	
		16600	.7486589619	.9999500647	

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COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS

TABLE IX

(1)

n = 31268	
cal.	exp.
73.9	
81.4	
88.3	
94.7	93.1
100.7	99.0
106.3	104.5
111.7	109.7
116.9	114.6
121.8	119.1
126.5	124.2
130.8	

(2)

n = 31268	
cal.	exp.
39.0	43.8
52.1	55.0
62.1	64.2
70.9	72.3
78.7	79.6
85.8	86.3
92.4	92.3
98.6	
104.3	

(3)

n = 31268	
cal.	exp.
53.2	
63.2	67.0
71.9	74.7
79.6	81.0
86.8	87.7
93.2	94.0
99.3	
106.7	
110.4	

(4)

n = 31268	
cal	exp.
87.3	
95.0	
101.0	
106.6	107.0
112.0	110.8
117.1	
122.0	120.3
126.7	125.0
131.3	128.8

(4′)

n = 31267	
cal.	exp.
28.77	[illegible]
56.31	56.5
65.8	65.7
74.2	
81.7	81.9
87.3	89.2

COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS

TABLE IX

(1)

n = 32768	
calc.	exp.
75.2	
81.4	
88.5	
94.7	92.1
100.7	98.0
106.7	104.0
113.7	109.7
119.9	114.9
126.0	119.1
132.0	124.2
138.0	

(2)

n = 65536	
calc.	exp.
35.0	43.0
41.1	48.0
47.1	53.0
53.1	58.0
59.2	63.0
65.2	68.0
70.9	73.2
76.7	78.0
82.5	
88.3	
94.2	
100.0	
105.8	

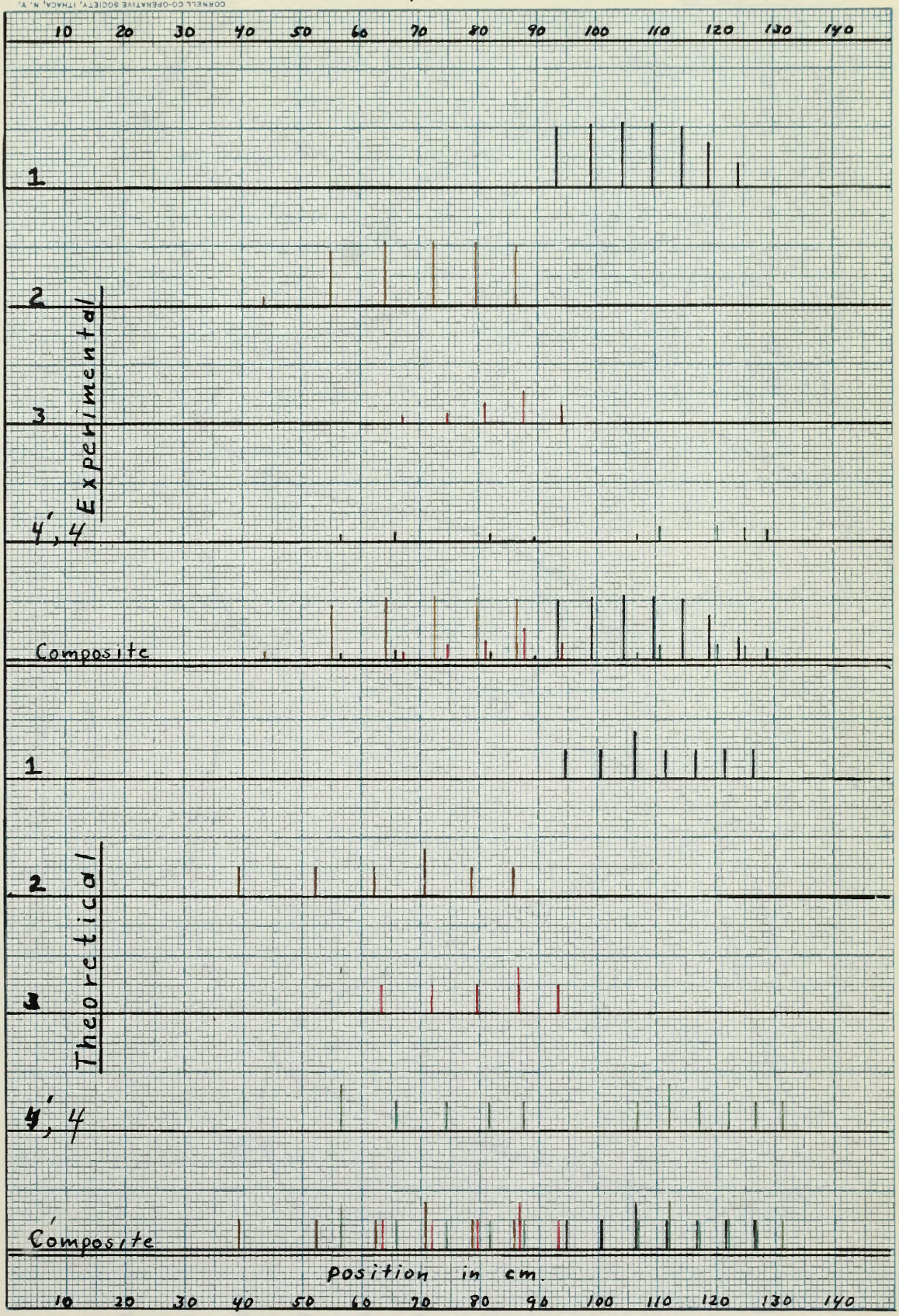
(3)

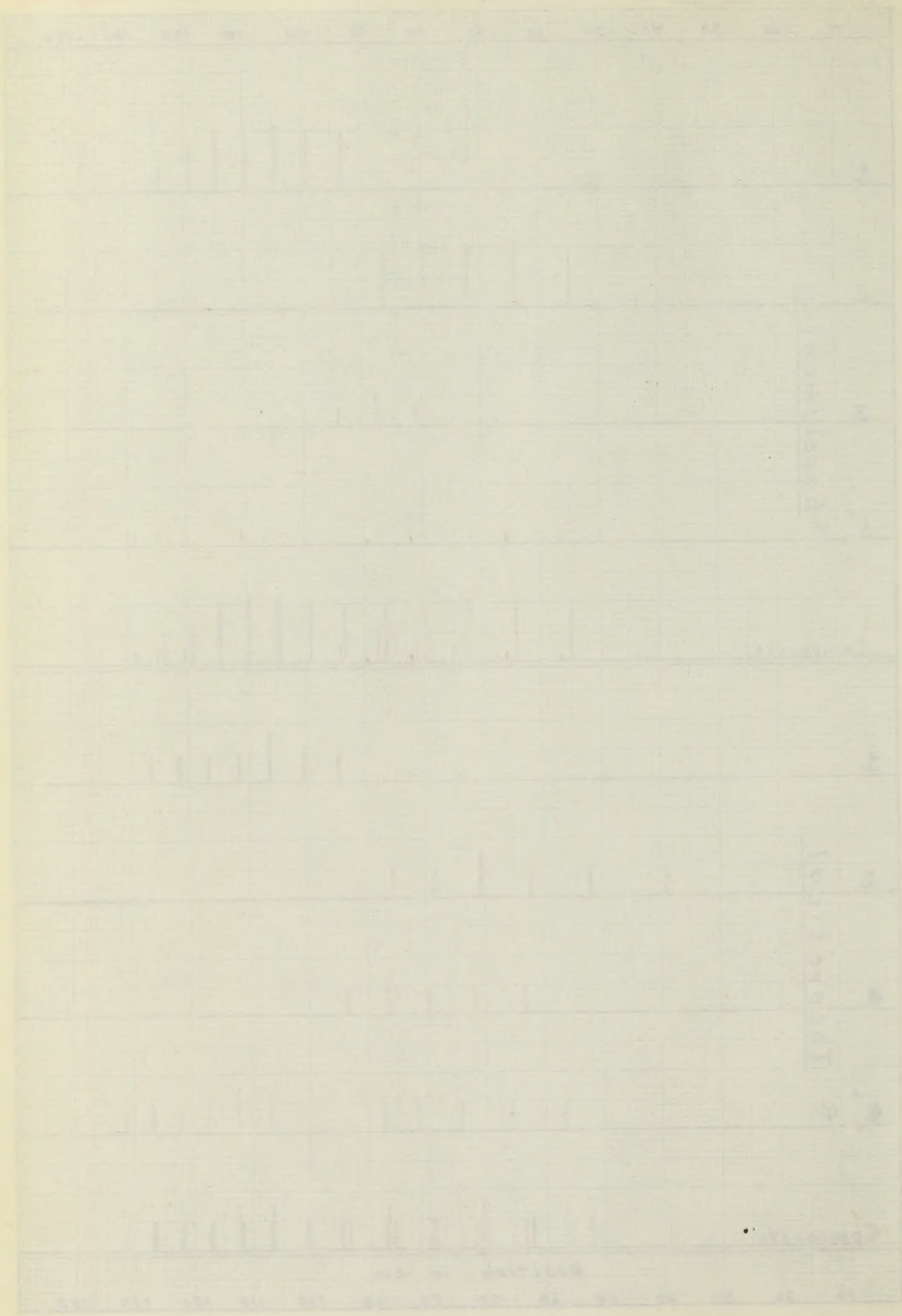
n = 131072	
calc.	exp.
37.5	
43.0	
48.5	
54.0	
59.5	
65.0	
70.5	
76.0	
81.5	
87.0	
92.5	
98.0	
103.5	
109.0	
114.5	
120.0	
125.5	
131.0	

(4)

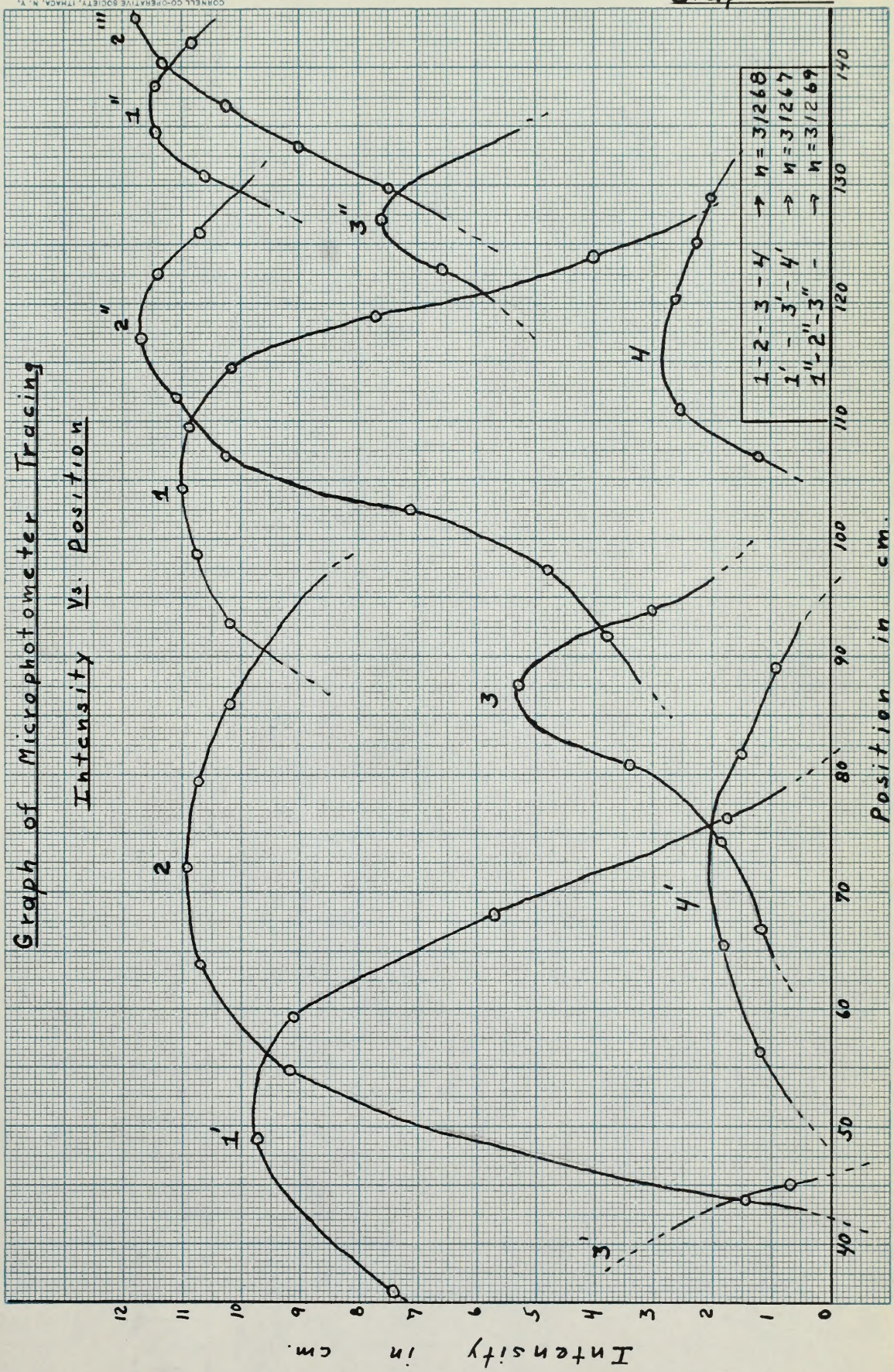
n = 262144	
calc.	exp.
37.5	
43.0	
48.5	
54.0	
59.5	
65.0	
70.5	
76.0	
81.5	
87.0	
92.5	
98.0	
103.5	
109.0	
114.5	
120.0	
125.5	
131.0	

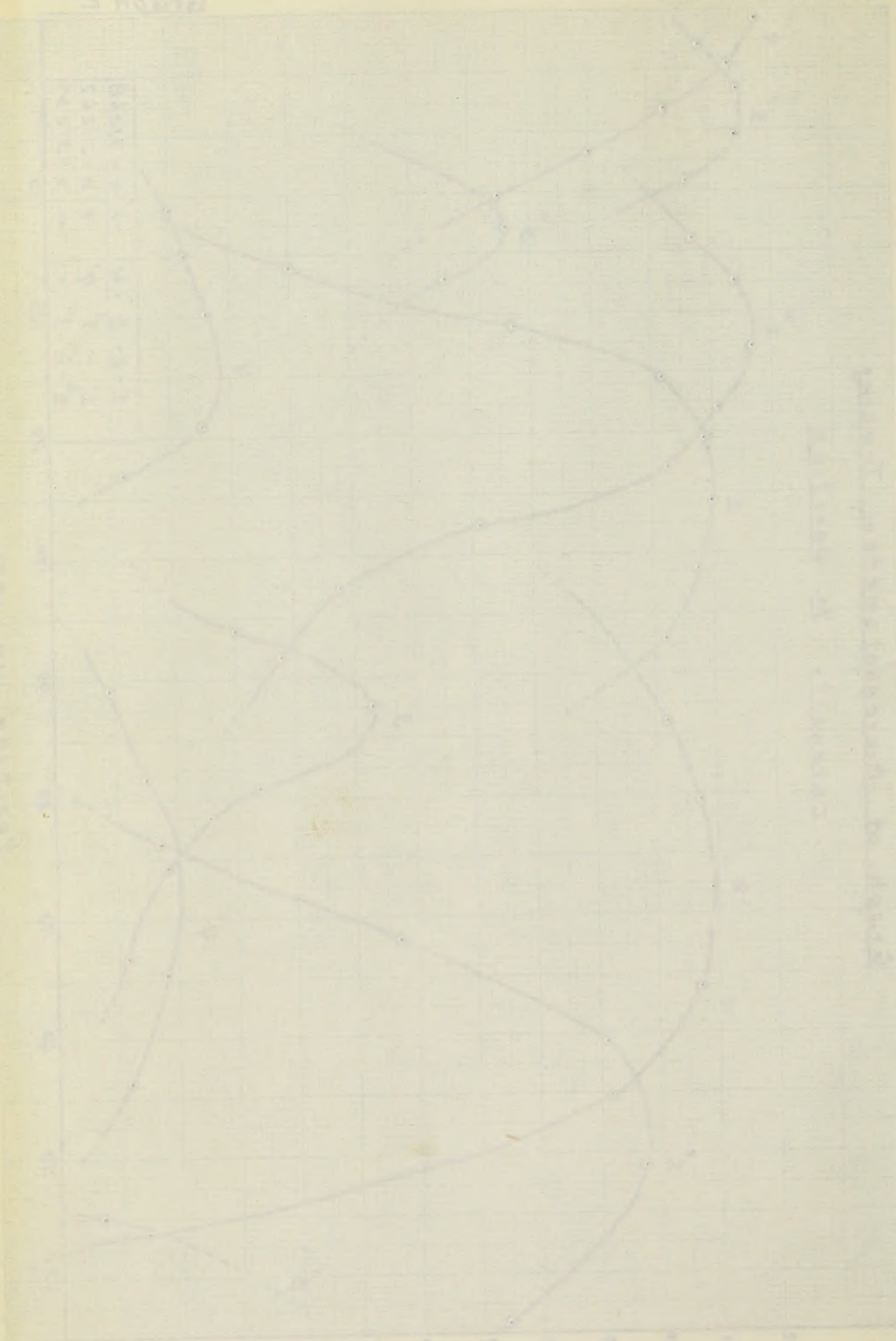
Graph 1

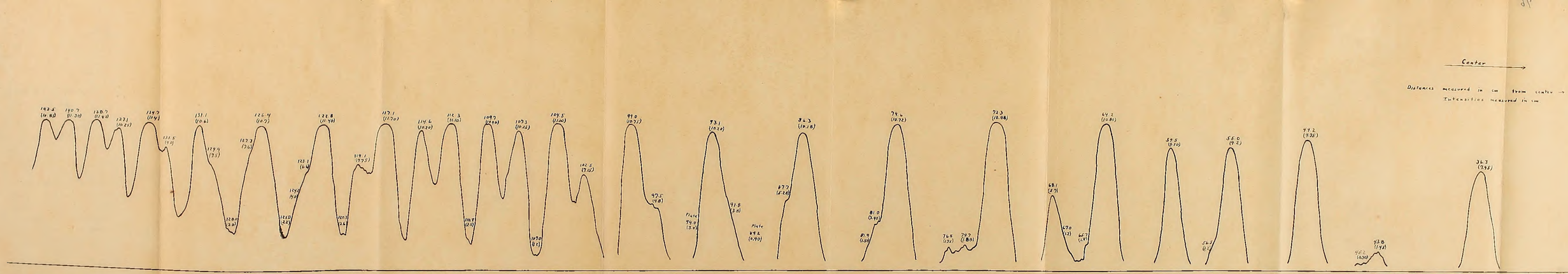

10 20 30 40 50 60 70 80 90 100 110 120 130 140
1
2
3
4', 4
Experimental
Composite
1
2
3
4', 4
Theoretical
Composite
position in cm.
10 20 30 40 50 60 70 80 90 100 110 120 130 140



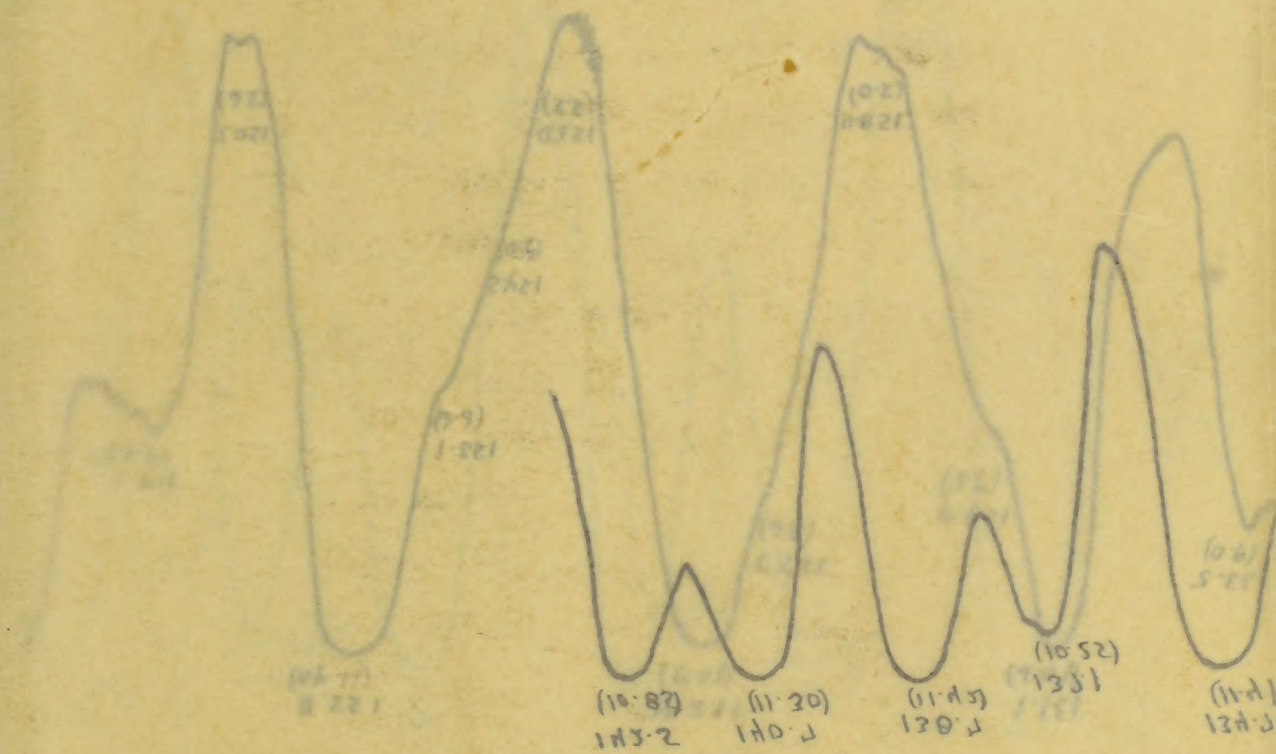
Graph 2


Graph of Microphotometer Tracing
Intensity Vs. Position
Intensity in cm.
Position in cm.
1-2-3-4' → n=31268
1'-3'-4' → n=31267
1''-2''-3'' → n=31269
1
2
3
4
1'
2'
3'
4'
1''
2''
3''
4''





Microphotometer Tracing of
Plate MS-3



DISCUSSION OF RESULTS

The theoretical calculations of the positions of the lines of the H-alpha complex obtained by the combination M S are found to agree closely with the positions of the lines determined experimentally from the microphotometer curve of the photographic plate MS-3.

The graph on page 75 shows the intensity of the various components of the patterns as a function of their distance from the center of the system. These lines fall into four distinct groups clearly indicating four components. Curves (1),(2),(3) and (4) belong to the order number 31268 and (1') & (4') to the order number 31267 of the first plate. Both the component lines and the maxima of the groups agree closely with the theoretical positions resulting from the constants of the Lummer plates, the angles of incidence and the angle between the Lummer plates.

The success achieved in checking so closely (to within 4%) the microphotometer curve by tracing the theoretical components of H_α through the two Lummer plates suggests that the problem could be worked backwards to determine the exact wavelengths and corresponding relative intensities. However, before this can be done other important factors must be considered. These will be discussed later.

Let us consider the two fundamental equations for the second Lummer plate.

$$\cos r_3 = \frac{\lambda}{2d'\mu'} n' \qquad \text{---} \quad A$$

$$\cos \psi_3 = \frac{\lambda}{2d'} n' \tan r_3 \qquad \text{---} \quad B$$

Since n' must be an integral number, then, for a given λ the angles r_3 and in turn the angles of emergence ψ_3 are fixed. If now the angle of incidence i_2 is varied slightly, then r_2, r_3 and consequently ψ_3 will change according to the equations —

$$\sin r_2 = \sin i_2 / \mu$$

$$r_3 = \beta' - \delta' + r_2$$

$$\cos \psi_3 = \mu' \sin r_3$$

Introducing these arbitrarily slightly different values of ψ_3 & r_3 in the formula

$$n'_{Max} = \frac{2d' \cos \psi_3}{\lambda \tan r_3} \qquad \text{---} \quad C$$

we can determine the value of the order number attending the direct ray ψ_3 (the most intense one). Thus if i_2 is changed sufficiently, we shall have a new order number n'_M, which means that the maximum intensity will shift along the fixed ψ'_3 for any given component as determined by equation B. This was checked experimentally. From a visual study of the pattern system as the second Lummer plate was rotated the maximum and minimum regions moved accross the field of view with the component lines remaining fixed. This means that the angle between the plates, the angles of the prisms and the angle of incidence need not be measured to a high degree of precision as was first thought necessary. These angles can be altered slightly to suit the problem at hand. This justifies the slight changes made in the angles in the theoretical calculation as previously mentioned.

The question to be asked is then---Why do not the experimental and theoretical positions of the lines check perfectly? The angles ψ_3 which determine these lines are governed by the equations A and B. Assuming now that the wavelengths and the corresponding μ'_4 are accurately known, then d' the thickness of the plate must be known to a high degree of precision. Thus, if work is to be continued in this field in getting accurate experimental values of the H_α components, the thicknesses of the Lummer plates must be known more accurately than as given in the section "Constants of the Lummer plates". Furthermore, the focal length of the telescope lens should also be determined more accurately for this controls the order number to be assigned to the most intense line of a component group of the final pattern when the problem is worked backwords.

The results of this research show definitely the existence of the third and the fourth components whose wavelengths are in approximate accord with the theoretical values.

CONCLUSIONS

To obtain better photographic plates of the H_α patterns given by two Lummer plates in tandem and thus obtain better microphotometer curves which might reveal the third and fourth theoretical components, a new optical arrangement was decided upon, as is described in the section "Optical System". In this arrangement the light from the first Lummer plate was directly incident on the second. With three Lummer plates of different dispersion and resolving power, six combinations of two plates each were possible. The patterns obtained by these combinations were photographed and studied. A detailed discussion of each pattern is given in the section "Spectrograms and Data". Many of these failed to yield the expected complex structure of H_α. The best result by far was the combination in which the medium plate was set first and the smallest plate last. A study of the microphotometer tracing obtained from this combination clearly indicated the presence of four components of the theoretical five component structure of H_α. The combination of the smallest plate and the largest also gave some structure indicating the presence of the third component but failed to show the existence of the fourth. This particular combination which was used by Robinson with a different spectroscopic system did, however, reveal the presence of the fourth component. It was apparent then that the present optical system showed lower resolving power than Robinson's, although the intensity of the transmitted light was increased sufficiently so that the visual study of the patterns was made easier. This reduction in resolving power was

due to the fact that some of the light coming from the first plate near grazing emergence did not fall upon the prism of the second plate. This diminished the effective length of the first plate and therefore its resolving power. Moreover, the combination (medium-smallest) which yielded the most interesting patterns in the present optical system is characterized by the least resolving power because of the small aperture of the smallest plate which was last in the train. The results suggest then that this combination should be tried with Robinson's spectroscopic system in which the full effective length of the first Lummer plate is utilized.

A study of the three Lummer plates separately showed that the new medium plate, bought purposely for this investigation, gave the poorest definition while the smallest gave the best. If, then, the medium plate is to be used at all, it would be best to use it as the first plate of the combination where the light coming from it acts as a source for the second plate.

It was found experimentally during the middle of the research that the light emerging from the plain side of a Lummer plate was more intense than that coming from the prism side due to the greater number of reflections of the beam before emerging. Accordingly the last group of photographs taken were made with this arrangement. This saved time in exposure. The plates were also placed such that their prisms were on opposite sides, thus partially compensating for the slight curvature imposed upon the line patterns which is inherent in a prism.

This pattern composed of perfectly straight lines gives better microphotometer curves because a finer and longer microphotometer slit can be used. The use of this type of slit gives smooth microphotometer curves eliminating plate grain effects and resulting in better resolution of the fine structure.

Neither of the microphotometers available, one at the Crufts Laboratory and one at the Harvard Observatory were suitable for the present problem. Besides the many mechanical difficulties which came up in preventing the running of the microphotometers, a great deal of time had to be spent in adapting a special lamp with a long filament, and additional lenses, to give a long, well defined image of the microphotometer slit on the photographic plate.

From the many photographs taken it was learned that for the best definition of the patterns the pressure and the current in the hydrogen tube had to be low. This reduced to a minimum the broadening of lines due to the Doppler effect. For best conditions it was found that the pressure should be about 0.9 - 1.2 mm of Hg with a current in the neighborhood of 20 milliamperes or a current density of 40 ma/cm^2.

Great difficulty has been experienced in this research in getting good photographic plates. At first films were used. These proved relatively fast, saving time in exposure and thus in cost of liquid air. However, they showed halation. Subsequently, plates were used and these were properly backed. Of the several types that were tried the majority of photographs

were taken with Eastman 4 1-C plates. These were none too satisfactory for they were never very clear, possibly due to chemical fog. The last few photographs taken were with the Eastman 144-F plates. These gave by far the best results.

Although the present research has fallen somewhat short of its objective, the presence of the fourth component of H_α has been indubitably proven. The constants of the Lummer plates have been determined and a means for obtaining the relative positions of these plates as well as the angles of incidence and emergence of the light rays. From a study of these angles and constants a method has been developed for the theoretical calculation of the positions of lines of the pattern system as obtained by two Lummer plates. This method has been applied successfully for one plate giving an agreement with experimental results, to within a maximum of 4%, establishing the existence of four components of the theoretical five component structure of H_α. With the suggestions as given in the section "Discussion of Results", the way appears clear for determining the exact wavelengths of these components.

In conclusion, the writer suggests a return to the spectroscopic system used by Robinson minus the *mirror* between the Lummer plates. His arrangement of the spectroscopic system should give greater resolving power. The best combination of plates is the medium plate first, followed by the smallest. The exclusive use of Eastman plates Type 144-F is advised. The overhead structure for photographing the optical

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*This was necessary because of cramped conditions in the research room.

system has already been built---anticipating the continuation of this investigation. Lastly, besides obtaining good photographic plates which reveal the fine structure components of H_α, a suitable microphotometer is absolutely essential.

ACKNOWLEDGEMENTS

The writer is grateful to Dr. Kent for having suggested this research problem, and for his invaluable assistance and time so generously given in all the investigations of an experimental nature. A like debt of gratitude is due Dr. Frye for his aid in the theoretical approach to this problem.

The writer also wishes to express his indebtedness to Dr. Shapley and Dr. Dimitroff of the Harvard Observatory for placing the microphotometer at his disposal, and to Mr. Albee for making the tracings.

BOSTON UNIVERSITY

GRADUATE SCHOOL

Dissertation

AN INVESTIGATION OF THE SEPARATIONS OF THE COMPONENTS OF THE H-ALPHA COMPLEX OF HYDROGEN

by

Thomas Homkowycz Wallace

(B.S., Boston University, 1933; A.M., Harvard University, 1936)

submitted in partial fulfilment of the

requirements for the degree of

Doctor of Philosophy

1939

(ABSTRACT)

The preliminary investigation of W. H. Robinson[1] in the study of H-alpha of hydrogen with the use of a Lummer plate as an auxiliary spectrograph appeared very promising. The source of radiation was a liquid -air cooled Wood's tube. Light from the tube passed through a filter which allowed only the H-alpha radiation to be transmitted. This radiation was brought to a focus on the slit of the first spectroscope that contained the first Lummer plate. The light emerging from this plate was reflected by a mirror* and then brought to a focus at the slit of the second Lummer plate spectroscope. Both plates were set to disperse horizontally. This method of using two Lummer plates in tandem furnished a new method for the study of fine structure. The results of Robinson's work showed the existence of a third component and indicated the presence of a fourth—— of the theoretical five component structure of H-alpha.

The aim of the present research was to extend Robinson's work in this field. It was hoped that with improved conditions and rearrangement of the spectroscopic system there could be obtained better photographic plates and thus better microphotometer curves which might more clearly reveal both the third and fourth components, especially the latter. With this in mind a third Lummer plate was bought for use in this problem.

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1 Robinson, W. H. PhD. Thesis, B.U. Grad.School, 1937

* This was necessary because of limited space in the research room.

The spectroscopic system finally decided upon was one in which the two Lummer plates were set directly in tandem with no intervening lenses. This allowed the emergent light from the first plate to be directly incident on the second plate. With less absorption of the light present, the intensity of the spectrum was increased enough so that the structure of the pattern could be studied visually more readily and exposure time somewhat reduced. From a study of the relative positions of these plates, their constants, and the angles of incidence and emergence of the light rays it was hoped that microphotometer curves might be obtained which would make possible a quantitative determination of both the wavelengths and intensities of the component lines of H-alpha.

The microphotometer tracing of one of the combinations of Lummer plates definitely showed the existence of four distinct groups of lines corresponding to four components of H-alpha. The constants of the Lummer plates have been determined and a means for obtaining the relative positions of these plates as well as the angles of incidence and emergence of the light rays. From a study of these angles and constants a method has been developed for the theoretical calculation of the positions of the lines of the pattern system as obtained by two Lummer plates. This method has been applied successfully for one Lummer plate combination. Both the component lines and the maxima of the groups were in agreement with experimental results to within a

maximum of 4% definitely establishing the existence of four components.

The success achieved in checking so closely the microphotometer curve by tracing the theoretical components of H-alpha through the two Lummer plates suggests that the problem could be worked backwards to determine the exact wavelengths and the corresponding relative intensities. Before this can be done, however, it will be necessary to obtain the thicknesses of the Lummer plates to a greater degree of precision, as well as to know more accurately the focal length of the telescope lens. A suitable microphotometer is also absolutely essential.

Comparison of the present spectroscopic system with Robinson's indicates that his should give greater resolving power. The combination of Lummer plates that has yielded the most structure of H_α in the present optical system is characterized by the least resolving power because of the small aperture of the smallest plate which was last in the train. The results suggest then that this combination should be tried with Robinson's spectroscopic system in which the full effective length of the first Lummer plate is utilized. The methods determined for the theoretical calculations of the final pattern can be easily adapted to this system.

Although, in this research, the wavelengths and the intensities of the components have not been determined, the presence of the fourth component ~~component~~ of H-alpha has been indubitably proven. It appears from the results of the present

investigation that the path is clear for determining the exact wavelengths and intensities. The present combination of the two Lummer plates (medium and small) has shown, by far, to be the best for revealing the fourth component and the possible detection of the fifth component if it exists.

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Born August 27, 1912 in Brighton, Boston, Massachusetts. Son of Anna Powlick and Wallace Homkowycz. Attended the Thomas Gardner Grammar School in Allston and the Mechanic Arts High School, both in Boston. Graduated from Boston University, College of Liberal Arts in 1933 with the degree of B.S. in Mathematics. Acted as an undergraduate assistant in the Physics Department during 1931-33. Entered the Harvard Graduate School in 1933 and received officially, the degree of A.M. in Physics in 1936. Attended the Boston University Graduate School in 1934 with appointment as a graduate assistant. Appointed to the Williams College Faculty in 1935 where the position of Assistant in Physics was held the first semester of that year. Was re-appointed to this position during 1936-1938. Member of the American Physical Society.

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